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MARKET EXPECTATION OF APPRECIATION OF THE RENMINBI

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Abstract

This paper proposes a path-dependent approach for estimating maximum appreciations of the renminbi expected by the market based on first-passage-time distributions. Using market data of the renminbi spot exchange rates, non-deliverable forward rates and currency option prices from 21 July 2005 (the reform of the exchange rate regime) to 28 February 2008 for model parameters, the maximum appreciations of the renminbi estimated under the proposed approach show that the market expected another large movement of the exchange rate during the 14 months after the reform. Subsequently, the few occasions of appreciations beyond the expected maximums coincided with trade-related issues and speculation that greater momentum of appreciation would be allowed by the authorities. The PBoC's measures were however largely incorporated into the derivatives' prices. The proposed approach can be used to gauge the range of appreciations of the renminbi anticipated in the market and to identify any exchange rate movements beyond market expectations.

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Executive Summary:

- *The People's Bank of China (PBoC) announced on 21 July 2005 a series of measures to reform the exchange rate regime, that included moving from a de facto peg to the US dollar to a managed floating exchange rate regime based on market supply and demand with reference to a basket of currencies, and a 2 per cent revaluation of the renminbi against the US dollar to CNY8.11 per US dollar.*

- *A sharp appreciation of the renminbi against the US dollar would have a negative effect on China's export-fuelled growth and might also cause a shock to the rest of Asia or even other developing economies. It is therefore an important issue to measure the range of appreciation expected in the financial market.*

- *This paper proposes a first-passage-time approach for estimating maximum appreciations of the renminbi expected by the market over a time horizon (i.e. not just at the end of it). Such path dependency is a critical factor that makes it possible to measure the extent to which the market has 'priced in' the possibility of substantial appreciation of the exchange rate triggered by an important economic-political event during the period of time.*

- *Using currency option prices which have the desirable property of being forward-looking in nature, the maximum appreciations of the renminbi estimated under the proposed approach show that the market prices incorporated the possibility of another large movement of the exchange rate during the first 14 months after the reform.*

- *Subsequently, the few occasions of appreciations beyond the expected maximums coincided with trade-related issues and speculation that greater momentum of appreciation would be allowed by the authorities. The PBoC's tightening monetary policy was however unlikely to be responsible for any unexpected acceleration of the exchange rate.*

- *The proposed approach can be used to gauge the range of appreciations of the renminbi anticipated in the market and to identify any exchange rate movements beyond market expectations.*

I. INTRODUCTION

The People's Bank of China (PBoC) announced on 21 July 2005 a series of measures to reform the exchange rate regime, which combined a revaluation of the renminbi and changes in the exchange rate mechanism. These included moving from a de facto peg to the US dollar to a managed floating exchange rate regime based on market supply and demand with reference to a basket of currencies, and a 2 per cent revaluation of the renminbi against the US dollar to CNY8.11 per US dollar. Under the new regime, the daily trading price of each basket currency against the renminbi will be allowed to move within a certain band around the previous day's closing price. The band width was undisclosed except for the US dollar, which is set at ±0.3% (then widen to ±0.5% on 17 May 2007), even though the PBoC emphasised that the band will change according to market development as well as the economic and financial situation. Included in the basket are the currencies of Mainland's major trading partners with weights reflecting to a large extent their trade shares.

With the growing importance of the Mainland China economy, the renminbi will gradually play a central role in international markets, and may become an anchor currency in Asia (see Greenaway et al. (2006) and Colavecchio and Funke (2007)). While the value of the renminbi under the new regime reflects the supply and demand, and can rise or fall against other currencies including the US dollar, appreciation of the renminbi has been a general trend since the reform and expected by market participants.[3] McKinnon (2006) shows that this expectation itself can influence China's economy even before substantial appreciation actually occurs. A sharp appreciation of the renminbi against the US dollar would have negative effect on China's export-fuelled growth. It would not just be Chinese companies that suffer as there would be a hit to margins for foreign companies with significant manufacturing exposure to China. A sharp appreciation may also cause a shock to the rest of Asia, which, although receiving a step gain in competitiveness, has been grateful for cheap Chinese imports. This may trigger substantial movements in the exchange rates of Asian economies or even other developing economies. It is therefore an important issue to measure the range of appreciation expected in the financial market.

Option markets have the desirable property of being forward-looking in nature and thus are a useful source of information for gauging market sentiment about future values of financial assets. Currency options, whose payoff depends on a limited range of the expected exchange rate, offer broader information about market expectations than the forward exchange rate.[4] The entire risk neutral probability density function of

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[3] The renminbi has appreciated about 12 percent as at February 2008 since the reform.

[4] The renminbi non-deliverable forward rates reflect the average expectation of the future exchange rate.

the exchange rate can be inferred from option prices. Campa et al. (1997) use currency option prices to derive market expectations of “effective” bandwidths for ERM currencies at the end of a time horizon based upon this kind of technique of extracting market expectations from financial prices. However, such bandwidths are determined by specifying the probability (say 90% or 95%) contained in the bandwidths. The specification is somehow arbitrary. In addition, such technique provides the expectations at the end of a time horizon (option expiration), i.e. a path-independent approach. Mizrach (1996) and Söderlind (2000) estimated the market expectation of the British pound exchange rate by using similar techniques for the ERM crisis period in 1992.

This paper proposes an approach for estimating the maximum appreciation of the renminbi expected in the financial market based upon a first-passage-time approach.[5] This approach is path dependent such that the characteristic of the exchange rate dynamics over a time horizon (i.e. not just at the end of it) is incorporated into the estimations. There is a significant difference between expected maximums measured by the path-independent approach and by the path-dependent approach. The measurement of the path-independent approach depends on the exchange rate only at the end of some time interval, and not on a particular path. This means that the path-independent maximum/minimum does not take into account the exchange rate reaching at a high/low level during some time interval. The path-independent approach therefore underestimates the maximum/minimum values (see Figure 1 below for illustration). In addition, it requires an arbitrary specification of the coverage (say 95%) of the probability density function to determine the maximum/minimum. This paper however considers that path dependency is a critical factor that allows substantial appreciation/depreciation of an exchange rate triggered by an important economic-political event during a time horizon. The expected maximum appreciation estimated by the first-passage-time approach could explicitly identify any exchange rate movements which are beyond market expectations reflected in financial prices.

In the following section, we derive the estimations of the expected maximum appreciation/depreciation of a currency whose exchange rate is under a lognormal process using the first-passage-time approach. In section 3, the expected maximum appreciations of the renminbi from 21 July 2005 to 28 February 2008 are calculated from the first-passage-time approach and the results are compared with actual exchange rate movements. The final section summarises the findings.

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[5] It should be noted that this paper is not about the equilibrium level of the exchange rate and whether the renminbi is undervalued.

II. FIRST-PASSAGE-TIME APPROACH

It is assumed that the spot exchange rate *S* (the renminbi value of a unit of the US dollar, i.e. CNY/USD) is under a lognormal process. The same assumption is used by the currency option valuation model in Garman and Kohlhagen (1983), which is the standard no-arbitrary model based on the framework developed by Black and Scholes (1973) for market participants to calculate implied option volatility.[6] The risk-free interest rates of the underlying exchange rate are constant. It is noted that the renminbi interest rate market has not been liberalised in China. The domestic renminbi interest rate is not tradable and does not reflect the forward exchange rate under the interest rate parity relationship that is a necessary condition in the Garman-Kohlhagen model. The non-deliverable forward (NDF) rate of the renminbi, which is traded in the off-share market, is thus used to imply the corresponding interest rate. The renminbi currency options are also traded in the off-share market.[7] Under the no-arbitrary environment in the Garman-Kohlhagen model, the Kolmogorov's forward equation governing the transition probability $P(S, t)$ of S is:

$$\frac{\partial P(S,t)}{\partial t} = \frac{1}{2}\sigma^2(t)S^2\frac{\partial^2 P(S,t)}{\partial S^2} - \left[r(t) - q(t) - \frac{\sigma^2(t)}{2}\right]S\frac{\partial P(S,t)}{\partial S}, \tag{1}$$

where r and q are the risk-free interest rates of the renminbi and US dollar respectively, and σ is the volatility of the exchange rate.

Let S_0 denote the exchange rate of the renminbi against the US dollar at current time $t = 0$. To estimate the expected maximum appreciation of the renminbi (i.e. the minimum exchange rate) in a time horizon of T, we define the following quantity:

$$S_{\min} = \min[S_t; 0 \le t \le T], \tag{2}$$

where $S_{\min}$ is the minimum for $0 \le t \le T$. $S_{\min}$ is estimated by considering the probability that $S_{\min}$ falls into a region between L and $\{L + \Delta L\}$, i.e. $\{L < S_{\min} < L + \Delta L\}$ for $0 \le t \le T$. This means that $S_{\min}$ breaches $\{L + \Delta L\}$ but not L. The movement of S under this representation is presented in Figure 1. The first-passage-time distribution of S is denoted by $D_{fp}(S,t)$. Therefore, the probability of $\{L < S_{\min} < L + \Delta L\}$ is:

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[6] While some studies suggest mispricing of currency options by the Garman-Kohlhagen model (see Ekvall et al. (1997)), the model is commonly used by market practitioners.

[7] According to BIS Triennial Central Bank Survey of Foreign Exchange and Derivatives Market Activity in April 2007, the daily average turnovers of currency forward and option transactions in the renminbi were US$4,572 million and US$244 million respectively (see Bank for International Settlements (2007)).

$$\begin{aligned}\Pr[L < S_{\min} < L+\Delta L] &= \Pr[S_{\min} < L+\Delta L] - \Pr[S_{\min} < L] \\ &= D_{fp}(L+\Delta L, T) - D_{fp}(L,T)\end{aligned}. \tag{3}$$

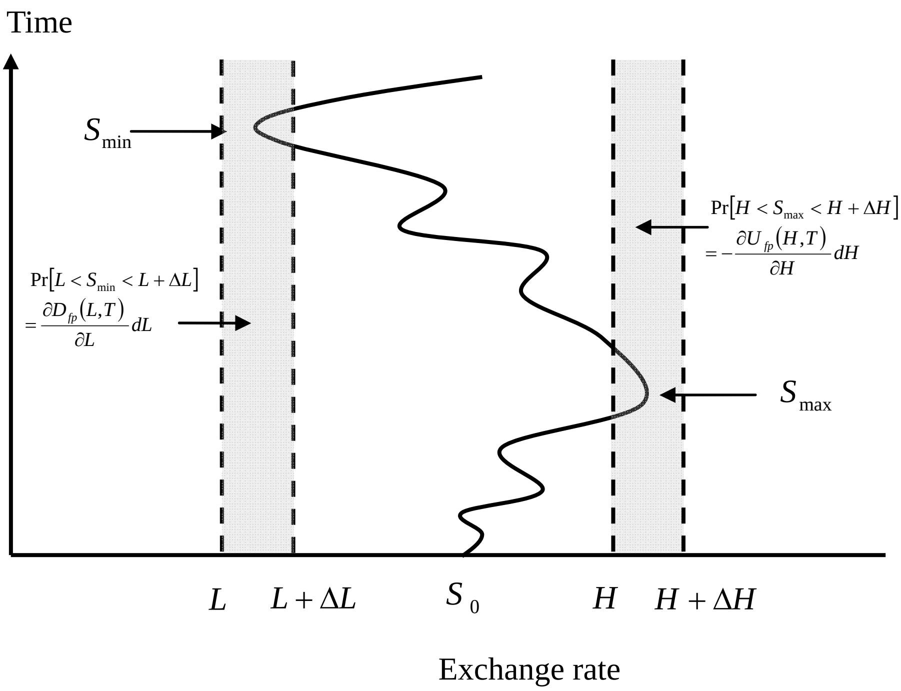
By taking a limit of $\Delta L \to 0$, Eq. (3) becomes

$$\Pr[L < S_{\min} < L+\Delta L] = \frac{\partial D_{fp}(L,T)}{\partial L} dL . \tag{4}$$

The expected minimum is thus given by

$$\langle S_{\min} \rangle = \int_0^{S_0} L \frac{\partial D_{fp}(L,T)}{\partial L} dL . \tag{5}$$

Figure 1. Schematic picture of exchange rate movement and its S_{min} and S_{max}



$D_{fp}(S,t)$ is derived from Eq. (1). After changing variable of $0 \le x \equiv \ln[S/L] < \infty$, the solution of Eq. (1) subject to the initial condition $P(x, t=0) = \delta(x - x_0)$ and natural boundary conditions is given by[8]

[8] $\delta(x - x_0)$ is the Dirac delta function.

$$P(x,t) = K(x,t;x_0,0) = \frac{1}{\sqrt{4\pi c_2(t)}} \exp\left\{ -\frac{[x - x_0 - c_1(t)]^2}{4c_2(t)} \right\} \tag{6}$$

with

$$c_1(t) = \int_0^t -\left[r - q - \frac{1}{2}\sigma^2 \right] ds$$
$$c_2(t) = \int_0^t \frac{1}{2}\sigma^2 ds \tag{7}$$

By the method of images, the corresponding first-passage-time probability density function with an absorbing boundary at $S = L$ (with $L < S_0$) is

$$P_{down}(x,t) = \int_0^\infty dx' \left[K(x,t;x',0) - K(x,t;-x',0)e^{-\beta x'} \right] \delta(x' - x_0) = K(x,t;x_0,0) - K(x,t;-x_0,0)e^{-\beta x_0}, \tag{8}$$

where

$$\beta = -\frac{c_1(t)}{c_2(t)}$$

and $x_0 \equiv \ln[S_0 / L]$.[9] Based on Eqs. (6) and (8), the first-passage-time distribution is given as

$$D_{fp}(L,t) = 1 - \int_0^\infty dx [P_{down}(x,t)] = 1 - \int_0^\infty dx \left[K(x,t;x_0,0) - K(x,t;-x_0,0)e^{-\beta x_0} \right] = N\left[\frac{-\beta c_2(t) - x_0}{\sqrt{2c_2(t)}} \right] + N\left[\frac{\beta c_2(t) - x_0}{\sqrt{2c_2(t)}} \right] e^{-\beta x_0} \tag{9}$$

where *N*(.) is the cumulative normal distribution function. By putting Eq. (9) into Eq. (5) with constant interest rates and volatility, the expected maximum appreciation $\langle S_{\min} \rangle$ is expressed as

$$\langle S_{\min} \rangle = S_0 \left(\frac{\beta}{\beta + 1} \right) N\left[\frac{\beta c_2(T)}{\sqrt{2c_2(T)}} \right] + S_0 \mathrm{e}^{(\beta+1)c_2(T)} \left(\frac{\beta + 2}{\beta + 1} \right) N\left[\frac{-(\beta + 2)c_2(T)}{\sqrt{2c_2(T)}} \right]. \tag{10}$$

The detailed derivation of Eq. (10) is in the Appendix.

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[9] The method of images is used to derive the closed-form formula for the first-passage-time density of a time-dependent Ornstein–Uhlenbeck process to a parametric class of moving boundaries in Lo and Hui (2006).

The corresponding maximum $S_{\max} = \max[S_t; 0 \le t \le T]$ (i.e. the minimum appreciation or maximum depreciation of the renminbi) is estimated by considering the probability that $S_{\max}$ falls into a region between H and $\{H + \Delta H\}$ (with $H > S_0$). Figure 1 shows that $\{H < S_{\max} < H + \Delta H\}$ for $0 \le t \le T$. The first-passage-time distribution of S is denoted by $U_{fp}(S,t)$. The probability of $\{H < S_{\max} < H + \Delta H\}$ is:

$$\Pr[H < S_{\max} < H + \Delta H] = -\frac{\partial U_{fp}(H,T)}{\partial H} dH . \tag{11}$$

The expected maximum is thus given by

$$\langle S_{\max} \rangle = \int_{S_0}^{\infty} -H \frac{\partial U_{fp}(H,T)}{\partial H} dH . \tag{12}$$

Similar to the derivation of $\langle S_{\min} \rangle$, the first-passage-time probability density function with an absorbing at $S = H$ is

$$\begin{aligned} P_{up}(\tilde{x},t) &= \int_{-\infty}^{0} d\tilde{x}' \left[K(\tilde{x},t;\tilde{x}',0) - K(\tilde{x},t;-\tilde{x}',0) e^{-\beta \tilde{x}'} \right] \delta(\tilde{x}' - \tilde{x}_0) \\ &= K(\tilde{x},t;\tilde{x}_0,0) - K(\tilde{x},t;-\tilde{x}_0,0) e^{-\beta \tilde{x}_0} \end{aligned} \tag{13}$$

with $0 \le \tilde{x} \equiv \ln[S/H] < \infty$ and $\tilde{x}_0 \equiv \ln[S_0/H] > 0$. Based on Eq. (13), the first-passage-time distribution is given as

$$\begin{aligned} U_{fp}(H,t) &= 1 - \int_{-\infty}^{0} d\tilde{x} \left[P_{up}(\tilde{x},t) \right] \\ &= 1 - \int_{-\infty}^{0} d\tilde{x} \left[K(\tilde{x},t;\tilde{x}_0,0) - K(\tilde{x},t;-\tilde{x}_0,0) e^{-\beta \tilde{x}_0} \right]. \\ &= N\left[\frac{\beta c_2(t) + \tilde{x}_0}{\sqrt{2c_2(t)}} \right] + N\left[\frac{-\beta c_2(t) + \tilde{x}_0}{\sqrt{2c_2(t)}} \right] e^{-\beta \tilde{x}_0} \end{aligned} \tag{14}$$

By putting Eq. (14) into Eq. (12) with constant interest rates and volatility, the expected minimum appreciation or maximum depreciation $\langle S_{\max} \rangle$ is expressed as

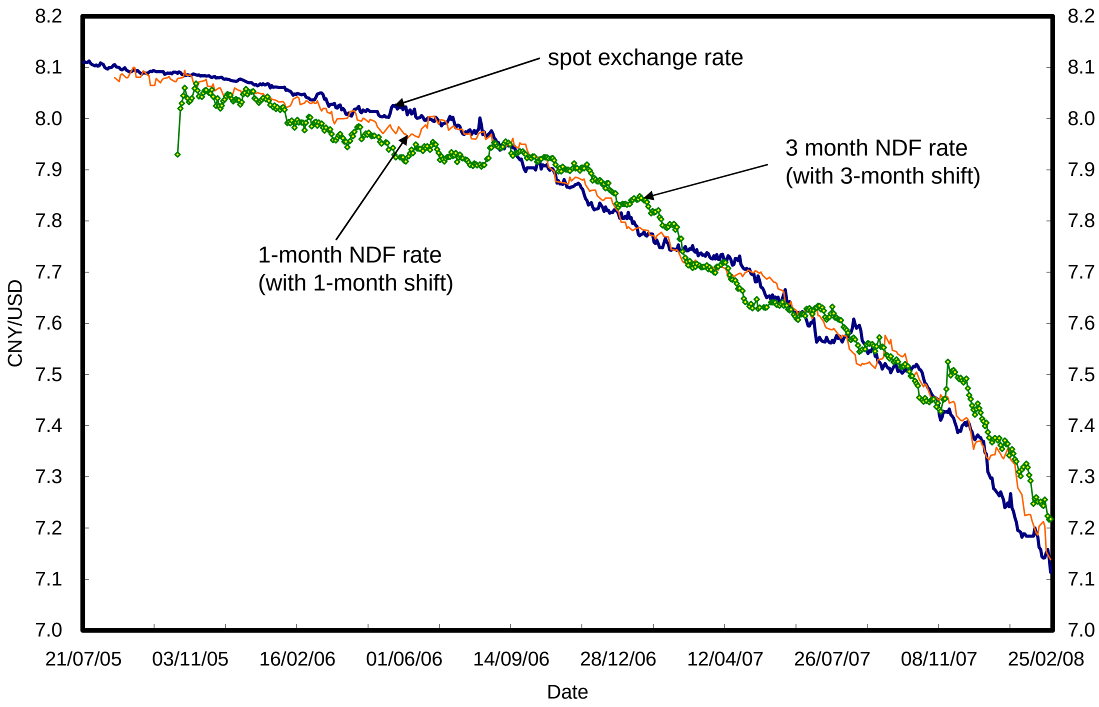
$$\langle S_{\max} \rangle = S_0 \left(\frac{\beta}{\beta+1} \right) N\left[\frac{-\beta c_2(T)}{\sqrt{2c_2(T)}} \right] + S_0 e^{(\beta+1)c_2(T)} \left(\frac{\beta+2}{\beta+1} \right) N\left[\frac{(\beta+2)c_2(T)}{\sqrt{2c_2(T)}} \right]. \tag{15}$$

The detailed derivation of Eq. (15) is in the Appendix.

III. Estimation of Expected Maximum Appreciation of Renminbi

The model parameters used to estimate expected maximum appreciation of the renminbi are spot exchange rates *S* (the renminbi value of a unit of the US dollar, CNY/USD), renminbi interest rates *r*, US dollar LIBOR *q*, and volatility σ implied from at-the-money-forward currency option prices covering the period from 21 July 2005 to 28 February 2008.[10] The NDF rates which are the average expected forward exchange rates of the renminbi are used to imply the renminbi interest rates *r*. Figure 2 presents the daily one-month and three-month NDF rates which are shifted one month and three months forward to compare with the actual spot exchange rates. It shows that continuous appreciations of renminbi were expected in the NDF rates while the actual exchange rates quite frequently appreciated more than the magnitudes implied by the forward exchange rates since September 2006.

Figure 2. Spot exchange rate, 1-month NDF rate (with 1-month shift) and 3-month NDF rate (with 3-month shift) of renminbi from 21 July 2005 to 28 February 2008



The estimated expected maximum appreciation $\langle S_{\min} \rangle$ and minimum appreciation (or maximum depreciation) $\langle S_{\max} \rangle$ of the renminbi with one-month and three-month time horizons based on Eqs. (10) and (15) are shown in Figures 3 and 4 respectively. The one-month and three-month expected maximum/minimum

[10] Data of option prices are provided by JPMorgan and other data are from Bloomberg.

appreciations ($\langle S_{min} \rangle$ and $\langle S_{max} \rangle$) are shifted one month and three months forward respectively to compare with the actual spot exchange rates. For example, the three-month $\langle S_{min} \rangle$ as at 1 June 2006 in Figure 4 is based on the market information as at 1 March 2006. It therefore takes three months to incorporate new market information into the input parameters. Figure 3 shows that the renminbi appreciated against the US dollar less than the expected maximums in one-month period in most of the observations, but well above the expected minimums. In particular, the expected maximums were substantially more than the actual appreciations of the renminbi from July 2005 until August 2006. This demonstrates that the market prices incorporated the possibility of another large (stepwise) movement of the exchange rate during this period after the 2 per cent revaluation on 21 July 2005. Since September 2006, the differences between the expected maximum appreciations and the actual exchange rates were narrower. This convergence may reflect that market participants had learnt and been getting used to the new exchange rate regime and the policies of the PBoC. Therefore, less risk premiums were priced into the currency option values. However, the renminbi strengthened beyond the expected maximums in some periods of time.

Figure 3. Spot exchange rate, 1-month expected maximum appreciation S_{min} and minimum appreciation (or maximum depreciation) S_{max} of renminbi from 21 July 2005 to 28 February 2008

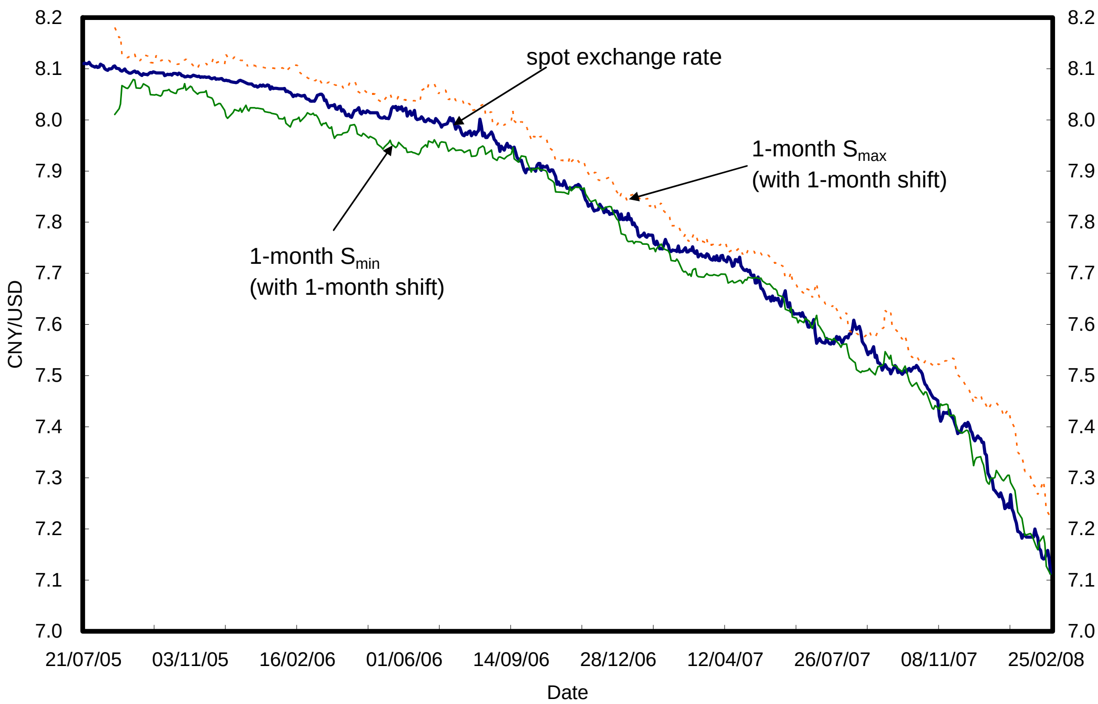


Figure 4. Spot exchange rate, 3-month expected maximum appreciation S_{min} and minimum appreciation (or maximum depreciation) S_{max} of renminbi from 21 July 2005 to 28 February 2008

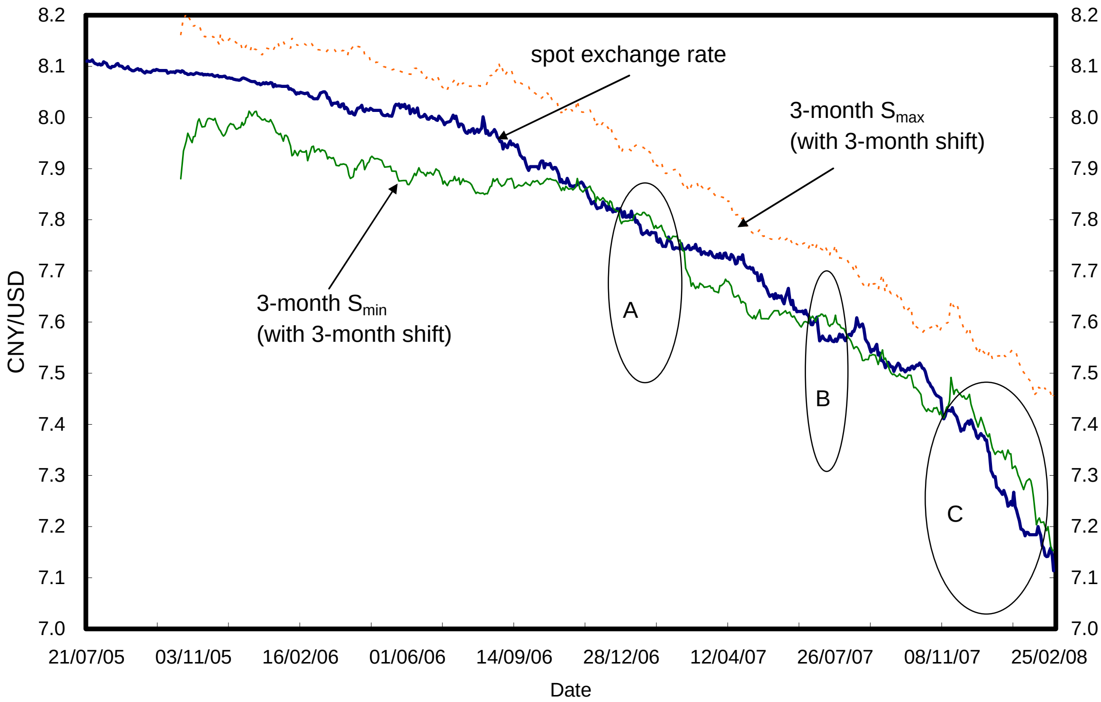


Figure 4 shows a clearer picture of such faster than expected maximum appreciations of the renminbi in three periods of time. The first period was at the beginning of 2007 (see period A in the figure). The spot rate climbed to RMB7.7436 per US dollar on 16 February 2007. The pickup in momentum in the appreciation appeared to be at least partly encouraged by the comments of the PBoC indicating that, while paying attention to inflation pressure, reform of the exchange rate regime would steadily advance and that the market would assume a greater role in determining the renminbi exchange rate. The second was in July 2007 (see period B in the figure) due to the higher than expected trade gap between China and the US that might sign faster appreciation allowed by the PBoC.[11] The third was between November 2007 and February 2008 (see period C in the figure). It is believed that the appreciation of the renminbi picked up further pace during this period was caused by calls for a faster appreciation of the renminbi by the European Union in the annual EU-China Summit in November 2007 and US negotiators in the third round of the Sino-US Strategic Economic Dialogue in December 2007 respectively. The calls tried to address China's

[11] China's trade gap widened 87 percent in June 2007 from a year ago to US$26.9 billion. A stronger yuan would increase export prices and lower import costs, reducing the surplus.

fast growing trade surplus.[12] In addition, the inflationary pressure in the first two months of 2008 added to the momentum.[13]

The exchange rate appreciations beyond the expected maximums in these three periods appear to coincide with the trade-related issues and speculations of greater momentum of appreciation allowed by the authorities. While the PBoC raised the reserve requirement ratio for deposit money banks 10 times from 9.5% to 14.5% and one-year deposit rate six times from 2.52% to 4.14% throughout 2007 (see Table 1), the tightening monetary policy was unlikely responsible for any unexpected acceleration of the exchange rate. Furthermore, the trading bands of the CNY/USD spot rate widened from 0.3% to 0.5% on 18 May 2007 but its impact on the exchange rate movement was not noticeable. This means that the market had already incorporated those PBoC's measures into the currency option prices and the NDF rates since the beginning of 2007.

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[12] China's trade surplus has been growing very quickly. The total trade surplus for the first 11 months of 2007 hit US$239.3 billion, far above the US$177.5 billion recorded for all of 2006. China's trade surplus with the EU rose 25 per cent in the first eight months of 2007 to €70 billon (US$103 billion).

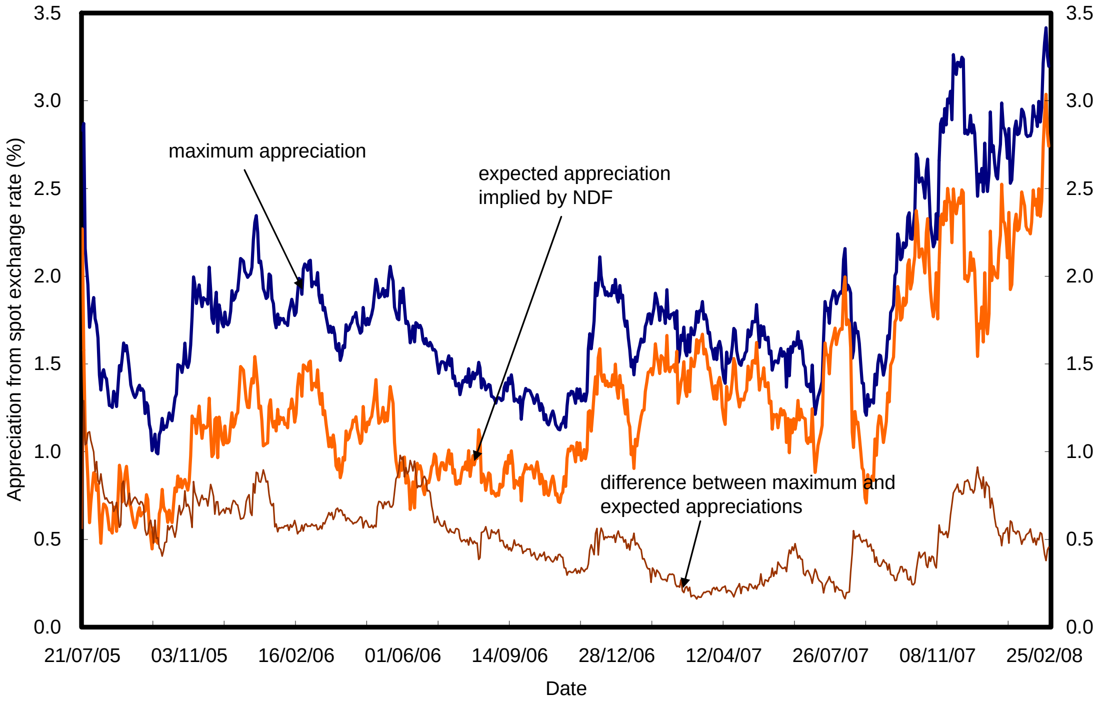
[13] The heavy snowstorms in early February 2008 pushed China's year-on-year CPI to 8.7% as at February 2008. The year-on-year CPI as at January 2008 was 7.1%.

Table 1. The BPoC's monetary measures in 2007

2007	
20-Dec	The PBoC raised the 1-year base lending rate from 7.29% to 7.47% and 1-year deposit rate from 3.87% to 4.14%. The base interest rates for other maturities were also raised accordingly, more notably in short term time deposits (e.g. 3-month base deposit rate rose by 45bps to 3.33%). However, the saving deposit interest rate was lowered by 9bps to 0.72%. All changes were effective from 21 December.
08-Dec	The PBoC raised the reserve requirement ratio for deposit money banks by 1ppt to 14.5% (effective from 25 December)
10-Nov	The PBoC raised the reserve requirement ratio for deposit money banks by 0.5ppt to 13.5% (effective from 26 November)
13-Oct	The PBoC raised the reserve requirement ratio for deposit money banks by 0.5ppt to 13.0% (effective from 25 October)
14-Sep	The PBoC raised the 1-year base lending rate from 7.02% to 7.29% and the 1-year base deposit rate from 3.60% to 3.87% (effective from 15 September)
06-Sep	The PBoC raised the reserve requirement ratio for deposit money banks by 0.5ppt to 12.5% (effective from 25 September)
21-Aug	The PBoC raised the 1-year base lending rate from 6.84% to 7.02% and the 1-year deposit rate from 3.33% to 3.60% (effective from 22 August).
30-Jul	The PBoC announced to raise the reserve requirement ratio for deposit money banks by 0.5 percentage point to 12.0% (effective from 15 August).
20-Jul	The PBoC raised the 1-year base lending rate from 6.57% to 6.84% and the 1-year deposit rate from 3.06% to 3.33% (effective from 21 July).
18-May	The PBoC announced to raise the reserve requirement ratio for deposit money banks by 0.5 percentage point to 11.5% (effective from 5 June). The PBoC raised the 1-year base lending rate from 6.39% to 6.57% and the 1-year deposit rate from 2.79% to 3.06% (effective from 19 May).
29-Apr	The PBoC announced to raise the reserve requirement ratio for deposit money banks by 0.5 percentage point to 11.0% (effective from 15 May).
05-Apr	The PBoC announced to raise the reserve requirement ratio for deposit money banks by 0.5 percentage point to 10.5% (effective from 16 April).
17-Mar	The PBoC raised the 1-year base lending rate from 6.12% to 6.39% and the 1-year deposit rate from 2.52% to 2.79% (effective from 18 March).
16-Feb	The PBoC announced to raise the reserve requirement ratio for deposit money banks by 0.5 percentage point to 10.0% (effective from 25 February).
05-Jan	The PBoC announced to raise the reserve requirement ratio for deposit money banks by 0.5 percentage point to 9.5% (effective from 15 January).

The differences between the expected maximum appreciations and the expected average appreciations implied by the NDF rates indicate the uncertainty of the exchange rate movements implied by the currency option market. Figure 5 shows the expected maximum appreciations, the expected average appreciations (as compared with the spot exchange rates) and their differences in a three-month horizon. The expected maximum appreciations were about 0.5% to 1% higher than the expectations implied by the NDF rates from July 2005 until end-December 2006. The differences then narrowed during the first ten months of 2007 while the expected appreciations increased. This reflects that the increase in the expected appreciation does not imply higher uncertainty anticipated in the market.[14] Given the tightening PBoC policy during 2007 and the wider trading bands, the market however priced less uncertainty into option prices over the same period. The differences increased again to the range of 0.5% and 1% in November 2007. This may be due to the high-level talks held between China and the US and also China and Europe and speculations of greater momentum of appreciation allowed by the authorities due to high inflation during the period. The results are consistent with those found in Figure 4.

Figure 5. Expected maximum appreciation, expected average appreciation (as compared with spot exchange rate) in 3-month horizon, and their difference from 21 July 2005 to 28 February 2008



IV. Conclusion

This paper proposes a first-passage-time approach for estimating maximum appreciations of the renminbi expected by the market. The characteristic of the renminbi exchange rate dynamics over a time horizon (i.e. not just at the end of it) is incorporated into the estimations. Such path dependency is a critical factor that allows substantial appreciation of the exchange rate triggered by an important economic-political event during the period of time.

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[14] The correlation between the expected appreciation and the difference is -0.2 over the sample period.

Using currency option prices which have the desirable property of being forward-looking in nature, the maximum appreciations of the renminbi estimated under the proposed approach show that the market prices incorporated the possibility of another large movement of the exchange rate during the first 14 months after the reform. Subsequently, the few occasions of appreciations beyond the expected maximums coincided with trade-related issues and speculations that greater momentum of appreciation would be allowed by the authorities. The PBoC's tightening monetary policy was however unlikely to be responsible for any unexpected acceleration of the exchange rate. The increase in the expected appreciation did not imply higher uncertainty anticipated in the market. The proposed approach can be used to gauge the range of appreciations of the renminbi anticipated in the market and to identify any exchange rate movements beyond market expectations.

APPENDIX

The expected minimum and maximum in Eqs. (5) and (12) are written as:

$$\begin{aligned}\langle S_{\min}\rangle &= \int_0^{S_0} L\frac{\partial D_{fp}(L,T)}{\partial L}dL \\ &= S_0 - \int_0^{S_0} D_{fp}(L,T)dL\end{aligned} \tag{A1}$$

and

$$\begin{aligned}\langle S_{\max}\rangle &= \int_{S_0}^{\infty} -H\frac{\partial U_{fp}(H,T)}{\partial H}dH \\ &= S_0 + \int_{S_0}^{\infty} U_{fp}(H,T)dH\end{aligned}. \tag{A2}$$

These two equations can be solved analytically by introducing $y=\ln[S_0/L]$ and $y=\ln[H/S_0]$ respectively. The integrations with respect to the hitting time distribution become

$$\begin{aligned}\int_0^{S_0} D_{fp}(L,T)dL &= S_0\int_0^{\infty}\left\{N\left[\frac{-\beta c_2(T)-y}{\sqrt{2c_2(T)}}\right]e^{-y} + N\left[\frac{\beta c_2(T)-y}{\sqrt{2c_2(T)}}\right]e^{-(\beta+1)y}\right\}dy \\ \int_{S_0}^{\infty} U_{fp}(H,T)dH &= S_0\int_0^{\infty}\left\{N\left[\frac{\beta c_2(T)-y}{\sqrt{2c_2(T)}}\right]e^{y} + N\left[\frac{-\beta c_2(T)-y}{\sqrt{2c_2(T)}}\right]e^{(\beta+1)y}\right\}dy\end{aligned} \tag{A3}$$

With constant input parameters, the integrals in Eq. (A3) can be expressed as the following form:

$$\begin{aligned}I(\theta,k,\xi) &= \int_{\theta}^{\infty} N\left[\frac{\xi c_2(T)-y}{\sqrt{2c_2(T)}}\right]e^{ky}dy \\ &= \int_{\theta}^{\infty} dy\int_{-\infty}^{z(y)} dz\,\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}e^{ky}\end{aligned}, \tag{A4}$$

where θ,k,ξ are constants and independent of y and

$$z(y)=\frac{\xi c_2(T)-y}{\sqrt{2c_2(T)}}.$$

By switching the order of the integration in dy and dz , Eq. (A4) becomes

$$\begin{aligned} I(\theta,k,\xi) &= \int_{-\infty}^{z(\theta)} dz \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \int_{\theta}^{y(z)} dy e^{ky} \\ &= \int_{-\infty}^{z(\theta)} dz \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \frac{1}{k}\left(e^{ky(z)} - e^{ka}\right) \end{aligned} \quad \text{(A5)}$$

with $y(z) = \xi c_2(T) - \sqrt{2c_2(T)}z$ and

$$z(\theta) = \frac{\xi c_2(T) - \theta}{\sqrt{2c_2(T)}} .$$

By completing square on the variable z , Eq. (A5) becomes

$$\begin{aligned} I(\theta,k,\xi) &= \int_{-\infty}^{z(\theta)} dz \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \frac{1}{k}\left\{e^{ky(z)} - e^{ka}\right\} \\ &= \frac{1}{k}\int_{-\infty}^{z(\theta)} dz \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} e^{k\left[\xi c_2(T) - \sqrt{2c_2(T)}z\right]} - \frac{1}{k}\int_{-\infty}^{z(\theta)} dz \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} e^{ka} \\ &= \frac{e^{\left(k\xi + k^2\right)c_2(T)}}{k}\int_{-\infty}^{z(\theta)} dz \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left[z + k\sqrt{2c_2(T)}\right]^2} - \frac{e^{ka}}{k}\int_{-\infty}^{z(\theta)} dz \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \\ &= \frac{e^{\left(k\xi + k^2\right)c_2(T)}}{k} N\left[\frac{(\xi + 2k)c_2(T) - \theta}{\sqrt{2c_2(T)}}\right] - \frac{e^{ka}}{k} N\left[\frac{\xi c_2(T) - \theta}{\sqrt{2c_2(T)}}\right] \end{aligned} . \quad \text{(A6)}$$

By substituting Eq. (A6) into Eq. (A3), the expected minimum and maximum are expressed as:

$$\langle S_{\min} \rangle = S_0 - S_0\left\{I[0,-1,-\beta] + I[0,-(\beta+1),\beta]\right\} \quad \text{(A7)}$$

and

$$\langle S_{\max} \rangle = S_0 + S_0\left\{I[0,1,\beta] + I[0,(\beta+1),-\beta]\right\} \quad \text{(A8)}$$

respectively. Using the identity of $N[z] = 1 - N[-z]$, Eqs. (10) and (15) are obtained.

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