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VALUING VARIABLE ANNUITIES WITH GUARANTEED MINIMUM LIFETIME WITHDRAWAL BENEFITS

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ABSTRACT

Variable annuities with guaranteed minimum lifetime withdrawal benefits (VA/GLWB) offer retirees longevity protection, exposure to equity markets, and access to flexible withdrawals in emergencies. We model how risk-averse retirees optimally withdraw from the products, balancing returns and the embedded longevity protection. Assuming reasonable individual preferences, the resulting cash flow generates a Money's Worth Ratio of around 0.9 for our stylized VA/GLWB in the post-crisis product, considerably lower than what was offered prior to 2008. Sensitivity analyses with respect to portfolio choice, mortality, fees, and guaranteed withdrawal rates show that MWRs range from 0.80-1.0, with the portfolio choice making the biggest difference. For most parameter choices, the utility value of the VA/GLWB exceeds that of a similar mutual fund, but it is less than for a fixed annuity. Interestingly, VA/GLWB withdrawals in early retirement can optimally exceed contract maximum withdrawals, despite the fact that this reduces future withdrawal guarantees.

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Variable annuities (VAs) offer retirees the security of a life-long retirement income stream along with access to the capital market via an investment portfolio. Though some critics have cited complexity and high fees as disadvantages of VAs,[1] these payout products are very widespread in the United States. For instance, policyholders held $1.6 trillion in VAs in 2012, and new sales were more than double the volume of fixed annuity sales (in 2011; $36.2 billion versus $16.9 billion; IRI 2012). Almost half of the money in VAs belongs to Baby Boomers, making this cohort the largest holder of VAs, followed by current retirees (35%; IRI, 2010). The decline of defined benefit plans in favor of defined contribution plans over the last half-century leaves this generation particularly vulnerable to longevity risk. VAs can serve as a substitute form of longevity protection, overcoming the perceived inflexibility of traditional fixed annuities. This, along with the fact that such a large share of the older population is heavily invested in VAs, prompts our interest in understanding how to value these products.

Most VA insurers include certain guarantee features in addition to a benefit payout stream. One very popular option, on which we focus in this paper, is the Guaranteed Lifetime Minimum Withdrawal Benefit rider (GLWB). This product provides the buyer with lifetime income benefits while also allowing him to take flexible withdrawals. Accordingly, the retiree may withdraw a certain percentage of his guarantee base if he wishes, where the base is usually specified as the initial purchase amount. Moreover, withdrawals in excess of the guaranteed amount are feasible, though these reduce the guarantee base. In this paper, we investigate how a risk-averse retiree can best trade off the complex product features; moreover, we measure the value of a GLWB for such a retiree. Today GLWBs are by far the most popular rider for VAs with living benefits. Most (84%) of all VAs held today include living benefits; three-quarters of net premiums are dedicated to GLWBs (Q4: 2009); and more than 60% of all VA purchases include a GLWB (LIMRA 2010).

Despite their importance in the market, researchers have not yet determined whether and how risk-averse retirees might value GLWBs as they balance the longevity protection of a traditional annuity with potentially higher returns from capital market investments. Accordingly, we calculate optimal withdrawal strategies for a risk-averse retiree and develop two ways to assess the appeal of this product. One is the Money's Worth Ratio (MWR) measure for the VA/GLWB, based on an optimal withdrawal strategy that trades off returns and longevity risk.[2] The product's longevity protection implies that these measures depend on market parameters including volatility, as these impact withdrawal strategies for the risk-averse retiree. Determining the value of a VA/GLWB is challenging, as withdrawals depend on several product parameters as well as on individual risk preferences. We model a stylized VA/GLWB using assumptions based on our extensive market overview for actual VA/GLWB products. At the same time, we acknowledge the variety in the current marketplace which could lead to lower or higher MWRs. Furthermore, our work utilizes commonly used assumptions on individuals' consumption patterns and risk-taking behaviors. We also provide sensitivity analyses for a variety of assumptions having important impacts on the MWR.

How consumers value the VA/GLWB depends on their preferences and how they utilize the product. Accordingly, we do not undertake a risk-neutral valuation assigning a unique value to each product assuming perfectly replicable portfolios which most previous studies did for various optional VA benefits as discussed later. Rather, we first explain why risk-neutral valuation seems inappropriate for a VA/GLWB, and we then elaborate our valuation approach generating MWRs. Modeling individuals with particular preferences helps understand the appeal of VA/GLWBs to the consumer.

To evaluate how a risk-averse decision maker would appraise such a product, we follow prior research exploring both MWR and utility valuations of simpler annuity products.[3] Optimal withdrawal patterns with VAs prove to be complex, since each withdrawal may affect the retiree's account balance as well as his future guarantee value, which in turn

1 See for instance Smartmoney.com (2011), and Orman (nd). Recently Pfau (2011) has critiqued their costs and lack of inflation protection. Brown et al. (2013) note that many Americans find the annuity concept complex.

2 Note that we do not calculate the MWR based on a withdrawal strategy that maximizes risk-neutral option value, but rather we examine how a risk-averse individual would optimally utilize this product.

3 Economic analysis of fixed annuities was launched with Yaari's (1965) important study followed by (among others), Mitchell et al. (1999) and recently, Davidoff (2009) and Lockwood (2013). International analysis of fixed annuities includes James and Song (2001), Fong (2002), Doyle et al. (2004), and Fong et al. (2011).

shapes the range of possible future withdrawals. Additionally, the optimal withdrawal strategy for a risk-averse expected utility maximizer must account for the negative effect of consumption fluctuations against which the GLWB protects.[4] We show that people do not optimally exercise the systematic withdrawal option, but instead they use it as a hedge against extreme longevity. In addition, since the GLWB can induce excessive risk-taking by retirees, insurers must limit the amount of equity exposure in the VA.

Several prior studies have explored the benefits of fixed annuities for risk-averse retirees, but to date, few researchers have focused on how VAs can help individuals finance their retirement needs.[5] The existing literature on VAs mostly takes the perspective of the insurer. For instance, several previous studies have examined actuarial aspects of the products including the pricing and hedging of VA guarantees, such as the Guaranteed Minimum Income Benefit (GMIB) which provides lifetime income protection but generally does not allow flexible withdrawals in retirement. The Guaranteed Minimum Withdrawal Benefit (GMWB) feature does not include longevity protection, since withdrawals cease if the account value is zero and the guarantee value has been depleted. The Guaranteed Minimum Account Benefit (GMAB) option ensures that the retiree's account value does not fall below a certain threshold net of withdrawals; nevertheless, there is no lifetime income security (unless the account value is held in a traditional life annuity). Still another product on offer is the Guaranteed Minimum Death Benefits (GMDB), which secures a certain account value for the buyer's heirs in the event of his death.

Some analysts, including Bauer et al. (2008), developed actuarial pricing models to determine risk-neutral pricing of the types of guarantees often embedded in VA products. Milevsky and Posner (2001), Ulm (2006), and Milevsky and Salisbury (2008) focused on GMWB pricing, while Holz et al. (2008) and Shah and Bertsimas (2008) investigated whether GLWBs are priced actuarially fairly. Those studies differ from ours in that they explored risk-neutral pricing for additional or optional VA benefits by computing the expected costs of these optional benefits and transforming them into yearly account payments. By contrast, our goal is to investigate how risk-averse consumers will optimally balance the longevity protection and investment access inherent in VA/GLWBs, and how they will value these products compared to their observed market prices.

A few prior VA researchers have taken a demand-side perspective, including Ulm (2010) who examined the impact of policyholder transfer behavior on GMDBs, and Brown and Poterba (2006) who empirically explored policyholder characteristics. Bernard et al. (2014) investigate optimal surrender behavior within GMABs for risk neutral policyholders. Some authors have examined asset allocations in VAs,[6] and Mahayni and Schneider (2012) reported that the existence of a GMAB induced higher risk-taking by policyholders during the accumulation period. Several additional studies[7] looked at optimal withdrawal strategies for VA investors in a risk-neutral framework; Dai et al. (2008) also derived optimal behavior for withdrawing funds from a GMWB, while Kling et al. (2011) studied withdrawal behavior with GLWBs. The latter two studies used the withdrawal patterns they derived to determine pricing of these guarantees and insurer hedging patterns. In contrast to our work, both of the latter papers assumed risk-neutral policyholders making option value-maximizing withdrawals. Huang et al. (2014) investigated optimal initiation times of GLWBs for a risk-neutral policyholder, but the authors assumed that policyholders followed a deterministic withdrawal pattern once benefits are initiated. By contrast, in what follows, we relax that assumption.

VA/GLWB benefits and costs have also received increasing attention in the practitioner literature of late. For instance, Blanchett (2011) investigated whether the embedded longevity option in GLWBs would be used; he found that the probability was between 3%-8%. He also concluded that VAs offer a relatively inexpensive form of longevity protection. Blanchett (2012) reported that GMWBs can make sense for a significant share of retirees' investments. Nevertheless, this last author also assumed that retirees follow a deterministic withdrawal strategy.

4 The Money's Worth Ratio evaluates expected cash flows, while the expected utility approach evaluates expected cash flows as well as the volatility of these flows.

5 Prior research on variable annuities includes Chai et al. (2011), Horneff et al. (2007; 2010a and b), Maurer et al. (2013), and Milevsky and Posner (2001).

6 *See* Charupat and Milevsky (2002), Horneff et al. (2010a and b), and Mahayni and Schneider (2012).

7 See Dai et al. (2008), Chen at al. (2008), Kling et al. (2011), Bacinello et al. (2011), Piscopo and Haberman (2011), Forsyth and Vetzal (2012), Feng and Volkmer (2012), Yang and Dai (2013), and Huang et al. (2014).

Until the financial crisis in 2008, the academic community seemed to agree that GLWBs were underpriced (c.f., Milevsky and Salisbury, 2006; Chen et al., 2008; and Dai et al., 2008). It was argued that the industry charged fees too low to cover expected costs on GLWBs, assuming risk-neutral policyholders who maximized option values or using a deterministic withdrawal strategy. Our work differs from prior studies by modeling optimal withdrawal strategies for risk-averse buyers of the VA/GLWB product. That is, we do not assume that buyers maximize their expected cash flows; rather, since we focus on securing retirement income, we model risk-averse individuals who evaluate alternative portfolio options not only according to their expected returns but also according to their payoff volatility. Moreover, it now seems clear that the market for GLWBs has changed considerably following the financial crisis (c.f., Huang et al., 2014). In particular, benefit payouts have been reduced and riskiness in investment options has been limited. This makes it all the more important to assess the appeal of actual products using post-crisis market prices and current benefits. We do so following an optimal withdrawal strategy for a risk-averse retiree and evaluate the VA/GLWB product as a whole rather than investigating the GLWB benefit independent from the VA product. In other words, our approach allows for potential cross-subsidization between product elements. Having incorporated all these elements, we derive money's worth ratios that we believe are more sustainable from an industry perspective.

In what follows, we first describe our modeling approach and assumptions used in our simulations. Next we present and discuss results, comparing the VA/GLWB product with two alternatives: a single premium immediate annuity (SPIA), and a comparable investment portfolio with no longevity insurance. We conclude with discussion of implications for insurers and policyholders.

METHODOLOGY

In what follows, we employ an expected utility framework to show how a rational risk-averse decision maker might value the VA/GLWB product as a means to finance retirement consumption. Accordingly, the retiree is assumed to determine how much of his wealth will be consumed each period, with the remainder saved for later. Deriving optimal consumption paths in the non-VA environment requires standard dynamic stochastic programming. Deriving optimal withdrawals for the VA/GLWB alternative is more difficult, as the individual's future account values and future guarantee amounts are influenced by current consumption.[8] After we have modeled both, we compare consumer well being when he buys the VA/GLWB, versus the SPIA or an investment portfolio. To do so, we first determine optimal withdrawals for the non-insured investment, and for comparability, we assume that the same portfolio of investments alternatives is available and selected within and outside the VA. Next, we provide sensitivity analyses using a range of sensible parameters embedded in a lifecycle consumption/saving model.[9] Finally, we show how results vary depending on asset allocations inside and outside the VA/GLWB.

The Lifecycle Model. We set up a dynamic stochastic programming model where individuals maximize their expected utility[10] over *T* possible remaining periods of life. Given additively separable preferences which are constant over time characterized by a utility function *u* with *u'*>0 and *u''*<0, the maximization problem takes the form:

$$\max_{C_1,\ldots,C_T}\left[U(C_1,\ldots,C_T)=u(C_1)+{}_1p_xE(u(C_2))+\ldots+{}_rp_xE(u(C_T))\right]. \qquad (1)$$

Here C_t with $t\in\{1,2,\ldots,T\}$ denotes consumption at each point in time *t* and ${}_tp_x$ the probability that an *x*-year old individual lives at least another *t* years. The maximization problem is subject to a budget constraint defined as:

$$W_t - C_t \geq 0 \qquad (2)$$

8 Our analysis omits taxes as we focus on the retirement period where tax differentials between VAs and direct investments play little role.

9 See for instance Koh (1998), Viceira (2001), Gomes and Michaelides (2005), Chai et al. (2011), Maurer et al. (2013), and Horneff et al. (2009, 2010a and b).

10 Gao and Ulm (2012) and Moenig and Bauer (2012) use a similar approach to investigate GMDBs and GMWBs.

where W_t denotes overall wealth at t. According to (2), the retiree cannot consume more than his current wealth level at any point in time, i.e., he cannot borrow against future income. Cash not consumed at date t is invested in a mixed portfolio consisting of different assets that pay off according to a geometric Brownian motion process. Here the risky asset evolves according to:

$$W_{t+1} = (W_t\text{-}C_t)\exp\left(\left(\mu - \sigma^2/2\right) + \sigma Z_t\right)_t \tag{3}$$

where μ denotes the drift of the Brownian motion, σ the standard deviation of the chosen portfolio, and Z_t the underlying Wiener process.[11] We rule out negative consumption (so $C_t > 0$); initial wealth is assumed to be positive ($W_o > 0$) and never becomes negative (hence $W_t \geq 0 \quad \forall t \in \{1,\dots,T\}$ where T denotes the maximum years in retirement). For the base case, we assume that the individual receives no utility from wealth remaining on his death (subsequently we investigate sensitivity of results to a bequest motive).

Rewriting the optimization problem (1) by the appropriate recursive Bellman equation, we obtain:

$$V(W_t) = \max\{u(C_t) + {}_1p_{x+1}E(V(W_{t+1}))\} \tag{4}$$

subject to (2) and (3) and the wealth constraints. Here V denotes the value function, which traces the expected utility of wealth level W_t under the optimal consumption path. This leads to the standard lifecycle model for financing retirement where, in the first period, the individual weighs consuming now versus later, given initial wealth.[12] Non-consumed wealth is invested in a combined portfolio according to risk preferences, and the consumer anticipates earning a stochastic return having a known mean and standard deviation. Next period's resulting wealth is again allocated between consumption and saving. Accordingly, the optimal consumption path is described as follows:

$$C_t^* = \arg\left[\max\{u(C_t) + {}_1p_{x+t} \cdot E(V(W_t - C_t,))\}\right].$$

Adding a VA/GLWB to the analysis implies that the value function depends not only on actual wealth remaining, but also on the guarantee value at each time period (G_t). Accordingly, the optimization problem changes to:

$$V(W_t, G_t) = \max_{C_t}\{u(C_t) + {}_1p_{x+t} \cdot E(V(W_{t+1}, G_{t+1}))\}. \tag{5}$$

In the simplest case, which we refer to below as the "plain" VA/GLWB, the guarantee evolves as follows:

$$G_{t+1} = G_t \cdot (1 - \max(0, (C_t - G_t \cdot WA)/G_t)). \tag{6}$$

Here the guaranteed withdrawal amount is usually the guarantee value times a predefined yearly withdrawal percentage WA. If the retiree were to withdraw more than the guaranteed amount, his guarantee value is reduced on a *pro rata* basis by the ratio of the excess withdrawal to the current account value. When someone withdraws more than the guaranteed amount and thus reduces his account basis; this is termed an "excess withdrawal" in VA provider prospectuses.

In practice, GLWB providers often include additional features to make the guarantee value more attractive, including so-called 'step-ups/ratchets' and 'roll-ups'.[13] The step-up/ratchet boosts the guarantee if the account value grows to exceed the guarantee value at certain pre-specified dates (often the policy anniversary). The step-up/ratchet guarantee can be modeled by adjusting the guarantee process as follows:

$$G_{t+1} = \max(W_{t+1}, G_t \cdot (1 - \max(0, (C_t - G_t \cdot WA)/G_t))). \tag{7}$$

11 Brownian motion is a widely-used approach to model the capital market. Although this model did not perform well during the recent financial crisis, additional research is needed to evaluate how these products might perform in a more complex capital market environment, since the guarantee value can depend on market jumps.

12 Retirement is characterized as a period of zero labor earnings with a specific amount of initial wealth available to invest at the beginning of the period.

13 For more discussion on these features, see Kling et al. (2010).

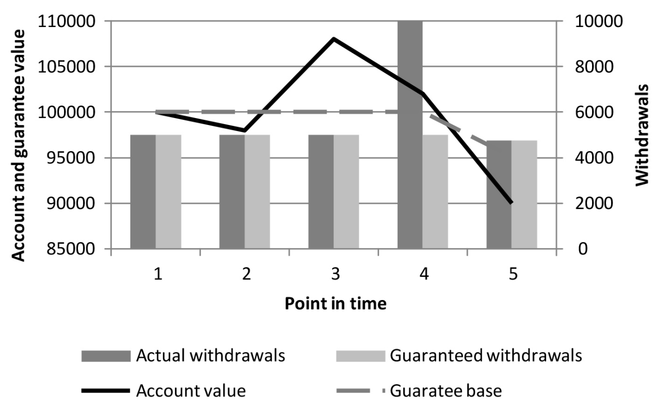
We refer to the latter case as the "ratchet" VA/GLWB. The roll-up provides for a periodic and specified interest rate increase on the guarantee value; for instance, a roll-up of 5% implies that the guarantee value increases by 5% per year. In this case the guarantee is as follows, where r denotes the roll-up interest rate:

$$G_{t+1} = G_t \cdot (1 - \max(0, (C_t - G_t \cdot WA) / G_t) \cdot (1 + r) \quad (8)$$

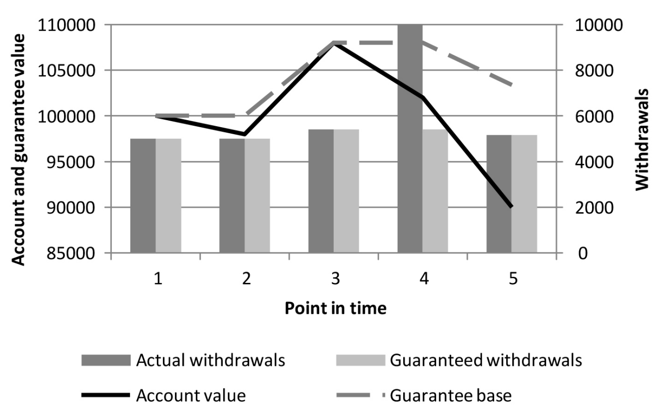
Figure 1 illustrates how the guarantee base G_t might evolve over time in a plain VA/GLWB and a ratchet VA/GLWB, respectively, where the initial account value W_0 and the guarantee base both equal $100,000. The example illustrates the case where a policyholder always withdraws the guaranteed withdrawal amount, except in the fourth period when he makes an excess withdrawal of $10,000. His account value peaks in period three due to favorable stock market developments. The left vertical axis depicts the guarantee base (dotted line) and account value (solid line) ranging from $85,000 to $110,000. The left vertical axis ranges from $0-$10,000 and displays actual (dark gray column) and guaranteed withdrawals (lighter gray column). Under the plain VA/GLWB, the favorable asset development in period three does not have an impact on the guarantee base, while it significantly increases the guarantee base under the ratchet VA/GLWB. The excess withdrawal in the fourth period decreases the guarantee base significantly under both alternatives. Nevertheless, the guarantee base under the ratchet VA/GLWB still exceeds the initial guarantee of $100,000 due to the earlier step-up.

FIGURE 1. HOW THE GUARANTEE DEVELOPS FOR A DETERMINISTIC EXCESS WITHDRAWAL PATTERN: TWO ILLUSTRATIVE CASES

A: Excess withdrawal in period 4: Plain VA/GLWB



B: Excess withdrawal in period 4: Ratchet VA/GLWB



Note: Panels A and B provide an example of how an excess withdrawal impacts the guarantee base under the plain and ratchet VA/GLWBs. We assume a deterministic withdrawal behavior, where the individual withdraws the guaranteed amount at times 1, 2, 3 and 5 and makes an excess withdrawal of $10,000 at time 4. The dark grey column shows actual withdrawals, and the light grey column the guaranteed withdrawals. The dotted line displays the guarantee base while the solid line illustrates the account value which changes due to withdrawals and asset performance.

In such a setting, optimal consumption from the VA/GLWB is determined by counterbalancing the effects of current consumption against the future account value; the latter may be used both to protect future consumption and future guarantees. Accordingly, the optimal consumption path is as follows:

$$C_t^* = \arg\left[\max\left\{u(C_t) + {}_1p_{x+t} \cdot E(V(W_t - C_t, G_{t+1}(C_t)))\right\}\right]. \tag{9}$$

In each period, the rational individual will determine whether the value function in the next period would be greater from consuming just the guaranteed withdrawal amount, versus consuming more or less. Even though the future guarantee value is reduced by doing so, consuming more than the guaranteed withdrawal amount could be optimal if the guaranteed withdrawal amount is relatively small compared to the current account value. Consuming more could also be appealing at very old ages, because there is less uncertainty about remaining life expectancy; in that case, the guarantee becomes less valuable even for a risk-averse individual.

Money's Worth Ratios. Two different approaches have been used to value annuity payout streams, with the first using risk-neutral pricing, and the second using real-world pricing. The first assumes the existence of a complete and arbitrage-free capital market which can be used to replicate payout streams from the VA. Accordingly, the embedded option of the additional guarantee has a unique price dictated by the replicating portfolio irrespective of policyholder risk preferences or the underlying investment portfolio within the VA. The price thus obtained equals the costs of the insurance company to hedge the guarantees. For GMABs[14] and GMDBs[15], for instance, options are either in the money or not at maturity or at the time of death, respectively. Exercise behavior should not depend on the policyholder's risk preferences and the payout stream can be replicated in the stock market. A GMWB is also a purely financial product in the sense that it does not provide longevity protection. Accordingly, policyholder preferences can impact withdrawal behavior for the product, but in practice this has little effect on withdrawals. Moenig and Bauer (2012) show that withdrawal patterns under risk-neutral versus real-world simulation scenarios do not differ substantially. These findings are consistent with FAS 122 which requires market-consistent pricing for GMABs and GMWBs. By contrast, below we note that GLWBs are perceived by consumers very differently from GMWBs. In particular, assuming a risk-neutral preference set is inadequate to capture actual risk-averse policyholder tastes when people value longevity protection.[16]

Accordingly, a second approach has been recommended in cases when consumers can elect when to take cash out of their VA product. For example, from the perspective of a risk-averse retiree for whom longevity protection is worthwhile, the Money's Worth he receives from the product will depend on his risk preferences and time preferences. Due to the strong dependence on consumer behavior and the emphasis on longevity protection within the product, real-world pricing can be used to determine the MWR of the product as commonly done for fixed annuities. This approach is a departure from a risk-neutral pricing as it adopts risk-adjusted discount rates as done for fixed annuities (accounting for insurers' potential default risk).[17] Actuaries have recommended this tack when exercise behavior importantly depends on policyholder preferences.[18]

The GLWBs on which we focus here differ from the other additional VA benefits discussed above, as they combine longevity protection with the possibility to receive a flexible pay-out stream. Previous work on GLWBs has used risk-neutral valuation to determine the value of a GLWB, but these studies have either modeled either static withdrawals (Blanchett 2011; Piscopo and Haberman 2011) or risk-neutral withdrawals (Kling et al. 2011). Modeling policyholder behavior based on risk-neutral presents the GLWB as a pure financial bet, affording no value to the product's embedded longevity protection.

14 For instance, Mayhani and Schneider (2011) simulate GMABs in a risk-neutral set-up.

15 See e.g. Milevsky and Posner (2001) for risk-neutral pricing of GMDB.

16 For this reason we do not compare GMWB results to GLWBs using risk-neutral pricing, since doing so would imply that longevity protection has zero value. Nevertheless risk-neutral simulation for GLWB is useful for insurers to determine pricing based on hedging costs.

17 See for instance Mitchell et al. (1999), James and Song (2001), and Fong et al. (2011)

18 For instance the Society of Actuaries notes (SOA 2005:13): "Financial economic theory tells you (to use) risk-neutral valuation for derivatives based on market prices. It doesn't talk much about policyholder behavior or the appropriate way to use policyholder behavior."

In what follows, we calculate the Money's Worth Ratio (MWR) for a VA/GLWB, assuming that the risk-averse retiree follows his optimal withdrawal strategy by trading off income and longevity protection. We determine how much such an individual would value such a product when used in the way that fits best his retirement needs. The MWR thus obtained will naturally be lower than a MWR where withdrawals are assumed to maximize expected cash flows in a risk-neutral world. This is because the risk-averse retiree is willing to forgo some monetary value to maintain insurance protection. In this way, our money's worth calculation takes into account the value of longevity protection.[19]

Since the VA/GLWB is a hybrid product including both an investment account and an annuity, one can think of dividing the benefit stream into two components, one resulting from the guarantee, and the other from payouts in excess of the guarantee. Under US regulation, the guarantee part is considered insurance, while cash withdrawn in excess of the guarantee is considered an investment product.[20] Accordingly, in what follows, we discount the portion of the withdrawal flowing from the guarantee using the corporate bond rate as this results from the insurance part of the product (c.f., Mitchell et al. 1999), and the investment portion using the net return for similar investments.

This also implies that, in the sensitivity analysis, the MWR varies with the drift . In a classical investment scenario, the MWR is assumed to be invariant to return changes since the expected return is used as the discount factor. Our case is different, since a portion of the withdrawals comes from the guarantee, i.e., the insurance component of the product. In sensitivity analysis, we show that changing the drift impacts withdrawal behavior which in turn influences which portion of the cash flow comes from the insurance element. Accordingly, here, the MWR varies with the drift.

In this setting, consumption flows arising from the guarantee and any remaining account value are discounted using a term structure of interest rates. When i_k denotes the interest rate in period k, the discount factor of one dollar invested at time j until time h is equal to $1/(1+i_{j+1})\cdot\ldots\cdot(1+i_h)$. Consumption over guarantee levels is discounted using the net return in the capital market. We define ${}_jP_x$ as the probability that an x-year old person survives another j years; then the MWR of an x-year old individual is equal to:

$$MWR(C_1,\ldots,C_T)=\sum_{j=1}^{T}\frac{(G_j\cdot WA)\cdot{}_jp_x+AV_j\cdot{}_{j-1}p_x\cdot(1-{}_1p_{x+j-1})}{\prod_{k=1}^{j}(1+i_k)}+\frac{(C_j-G_j\cdot wa)\cdot 1_{\{c_j>G_j\cdot wa\}}}{(1+\mu)^j}. \quad (10)$$

While the MWR is useful in determining the expected value of the cash flows per premium dollar spent, the concept does fully measure how a risk-averse decision maker will value the product taking into account investment risk as well as guarantee levels. Accordingly, below we compare MWR measures with utility measures as defined in (1), for those who purchase such a product. If the retiree dies before his account value is exhausted, the standard death benefit in a VA/GLWB is the remaining account value. In a base case, we abstract from bequests though we include remaining account balances at death in our MWR measures as this devolves to the buyer's account upon his death. In subsequent analysis, we also incorporate a bequest motive.

19 If the retiree dies before the account value is exhausted, the standard death benefit in a VA/GLWB is the remaining account value. Note that this differs from a fixed annuity where assets attributable to someone dying early are captured by the insurance pool and used to pay those who live longer than expected. In our base case, below, we abstract from bequests though we include remaining account balances at death in our MWR measures as this devolves to the buyer's account upon his death. In subsequent analysis we also incorporate a bequest motive.

20 Since the account balance is regarded as a security, it is regulated by the US Securities and Exchange Commission (SEC) and backed by the Securities Investor Protection Corporation (SIPC). By contrast, the VA insured guarantees are regulated by state Insurance Commissioners and backed by state insurance guarantee funds.

CALIBRATION

To generate a range of consumption outcomes for the VA/GLWB, we use Monte-Carlo simulation incorporating reasonable assumptions about preferences, expected risky returns, volatility, and VA parameters. Our choices for these are drawn from market evidence and the research literature, as described next.

Individual Parameters. We assume that a single 65-year old male enters retirement facing a mortality table defined by the 2000 Annuity Basic Mortality Table.[21] We use the Basic Table as it does not include margins or safety loadings, and we subsequently compare results with those using the U.S. Social Security Administration's 2006 cohort life table.[22] In addition, we assume that the average individual receives Social Security benefits of about $13,000 per year,[23] and he holds $100,000 in additional wealth at retirement with which he either buys a VA/GLWB or invests in capital market assets.[24] Cash-flows from these holdings are paid in addition to Social Security benefits. Following the standard lifecycle approach, we use an isoelastic utility function $u(c) = \frac{c^{1-RRA}}{1-RRA}$ with a time discount factor β=0.96 and RRA=5.[25]

As noted previously, our model omits taxes, mainly because we consider only immediate (and not deferred) VAs. Moreover our representative consumer is a median retiree for whom Social Security benefits are just sufficient to lift him above poverty. Accordingly, the additional benefits flowing from the VA are just enough to provide a modest life-style, and income taxes for such individuals will be minimal. As shown below, individuals do not accumulate much interest on gains over the years, so there would be little advantage from deferred taxation of capital gains.

Capital Market Parameters. To compare the returns from the two different portfolios, we assume that the individual holds the same mix of capital market assets irrespective of whether he buys a VA/GLWB or invests outside the VA. (Below our sensitivity analyses also allow for different market environments and investment strategies.)[26] Accordingly, in both cases, our retirees select portfolios similar to those held by real-world VA investors (IRI 2010): 48.5% in equity, 22.2% in fixed income, 14.7% in balanced funds/hybrids, 11.5% in bonds, and 3.3% in money market assets. Assuming a 10% return on equity, 6% on bonds, and 3% on safe investments, the average expected return gross of fees in the VA is then around 6.75%. Accordingly, the portfolio is modeled with μ=0.0675 and σ=0.18.[27] (Sensitivity analysis varying μ and σ appears below). Taking into account average fees for retail investments, we compute an annual investment charge of 1.26% for the average portfolio held outside the VA (IRI 2010).

To determine the optimal strategy for the non-VA investment, we use stochastic dynamic programming and compute the value function $V(W_t) = \max_{c_t} \{u(c_t) + E(V(W_t - C_t))\}$. We do this by discretizing the state space[28] and solve the problem by backward induction; at the last possible age T, it will be optimal to fully consume all remaining wealth (i.e. $c_T = W_T$). Using this information, we compute the values of the value function on the predefined grid; next, we back up a period and again determine optimal consumption for using information on the value function from the next period. This process is repeated until the first period is reached. Next we simulate 20,000 Monte Carlo paths for the portfolio and mortality processes. Optimal consumption for each path at each point in time is computed using the value function until all sample individuals are simulated to die. If sample individuals run out of assets when no longevity protection is purchased, we

21 SOA (1995) is the most recent annuity table widely used for pricing variable annuities. We also assume that anyone still alive at 110 years of age consumes all of his remaining wealth.

22 The 2006 cohort table available at http://www.ssa.gov/oact/NOTES/as120/LifeTables_Body.html is the most recent available.

23 This corresponds to average annual Social Security income in the U.S. (SSA 2011).

24 Poterba et al. (2012) report of $111,600 in median financial assets for a two-person household age 65-69.

25 Assumptions on beta and RRA follow Horneff at al. (2009), and Pang and Warshawsky (2010).

26 In principle, the consumer's portfolio allocation decision might also be endogenized, as VA products usually allow some choice over portfolio allocations. We do not do so here, since insurers generally limit portfolio options available to GLWB buyers, and most firms can also rebalance buyers' portfolios if the guarantee grows too costly. These restrictions lead us to adopt the average portfolio allocations found in real-world VAs. (Precise information on this portfolio rebalancing process is rarely publicly available; see Abbott et al., 2009).

27 Our parameters compare to those in Milevsky and Salisbury (2008), Maurer et al. (2010), and Kling et al. (2010); all are nominal.

28 For the grid choice, we use triangular numbers which implies a convex choice of grid points as a higher degree of precision is more important for lower values. Results are slightly improved when using triangular versus an equidistant grid.

assume that their only source of consumption is derived from Social Security benefits. Finally, using these simulated results, we calculate the MWR and expected utility for the non-VA investment.

VA Parameters. To model the annuity product, we note that yearly expenses for a plain VA with no additional guarantees averaged 2.43% in 2009 (IRI 2010); this total consisted of mortality and expenses (M&E), administrative and distribution charges, and investment management fees. The IRI (2010) also reports extensive data on fees by investment class, which we use to calculate a weighted VA annual fee of 2.47% of the account value (IRI 2010). In our calculation, 1.24 percentage points or 124 basis points (bps) are attributable to M&E fees, 18 bps to administrative and distribution charges, and the remaining 105 bps to investment fees. Thus investment management charges are below those for retail purchasers, but VA buyers do pay substantial M&E fees.[29] Information for 2010 is less extensive, but we do know that the average fee dropped from 243 to 233 bps, where M&E and administration fees accounted for 118 bps. Hence we adjusted the weighted investment charges by the same percentage decline, and we set the base case fee to 237 bps in the modeled VA. Since fees vary across insurers in the marketplace, in our sensitivity analysis we allow M&E and administration fees to vary from 129 to 280 bps.

To determine the additional cost of the GLWB, we reviewed online prospectuses for the 25 U.S. insurers having the most new VA sales in 2011. Four had no online prospectus; of the remaining 21 companies, five did not offer a GLWB, and one company offered only group VAs. Since ratchets are the most common GLWB enhancement, we restrict analysis to the plain guarantee as in (6) and the ratchet as in (7). The average fee for a GLWB providing a 5% withdrawal at age 65 was 0.98%, close to the fee average of 0.99% in 2010 (Morningstar Annuity Research Center Report 2010). Only one company we found clearly indicated a price differential of 0.25% between a GLWB with, versus without, the step-up. Accordingly we set the cost of the plain GLWB at 0.75% price and for the step-up GLWB feature at 1%.

It is evident that the marketplace offers a wide variety of VA/GLWBs, making it difficult to characterize any one of them as exactly 'typical.' We offer in Table 1 a summary of the 10 most popular VA products in the U.S. market, along with short product descriptions. The parameters employed in our analysis closely imitate many of the available products: for instance, the 5% withdrawal rate and pro-rata reduction in case of excess withdrawals are quite common (only a single company offered a guaranteed withdrawal of 4.5%). Below we offer sensitivity analyses regarding product prices and investment portfolios.

TABLE 1: THE 10 TOP SELLING VARIABLE ANNUITIES BY ASSET SIZE

Product name	Insurance company	Asset in 1,000 million $ (12/31/2013)	Fees	GWLB (name)	GWLB fees	GWLB withdrawal	Ratchet	Roll-up	Excess withdrawals
Perspective II (7-yr 2002)	Jackson Natl Life Insurance Co	61.65	1.25%	Yes (Lifeguard freedom 6)	2.52%	Earnings sensitive	Annually	None	Pro-rata reduction of guarantee base
Portfolio Director (Plus & 2)	Variable Annuity Life Insurance Co	52.20	1.25%	Yes (IncomeLOCK Plus)	1.10%	5.00%	Annually	6% (as long as withdrawals do not exceed 6% of guarantee base) for the first 12 years	Pro-rata reduction of guarantee base
Perspective L (2005)	Jackson Natl Life Insurance Co	31.75	1.60%	Yes (Lifeguard freedom 6)	2.52%	5%	Quarterly (locked in annually)	None	Pro-rata reduction of guarantee base

29 IRI (2010) indicates that M&E fees provide a VA buyer the option to annuitize the account value at a rate set at the beginning of the contract, a standard death benefit, and a promise that insurance charges will not increase.

Premier Retirement VA B	Pruco Life Insurance	29.02	1.30%	Yes (highest daily lifetime income 3.0)	1.00%	5%	Daily	5% (until withdrawals begin)	Pro-rata reduction of guarantee base
MetLife Investors Series VA	Metlife Investors USA Insurance Co	26.93	1.30%	No	N/A	N/A	N/A	N/A	N/A
Premier Retirement VA L	Pruco Life Insurance	20.75	1.70%	Yes (highest daily lifetime income 3.0)	1.00%	5%	Daily	5% (until withdrawals begin)	Pro-rata reduction of guarantee base
MetLife Investors Series L (4-yr)	Metlife Investors USA Insurance Co	19.26	1.60%	No	N/A	N/A	N/A	N/A	N/A
RVS RAVA4 Advantage (10-yr)	Riversource Life Insurance Co	18.44	1.05%	No new contracts issued	N/A	N/A	N/A	N/A	N/A
Venture III	John Hancock Life Insurance Co USA	17.30	1.65%	No longer available	N/A	N/A	N/A	N/A	N/A
Vision B	Allianz Life Ins Co of North America	16.83	1.40%	Yes(Income Protector - Single Lifetime Plus Payments)	1.20%	4.50%	Quarterly	7% for 30 years	Pro-rata reduction of guarantee base

Source: Morningstar sales data plus authors' summary of insurer prospectuses.

SIMULATION RESULTS

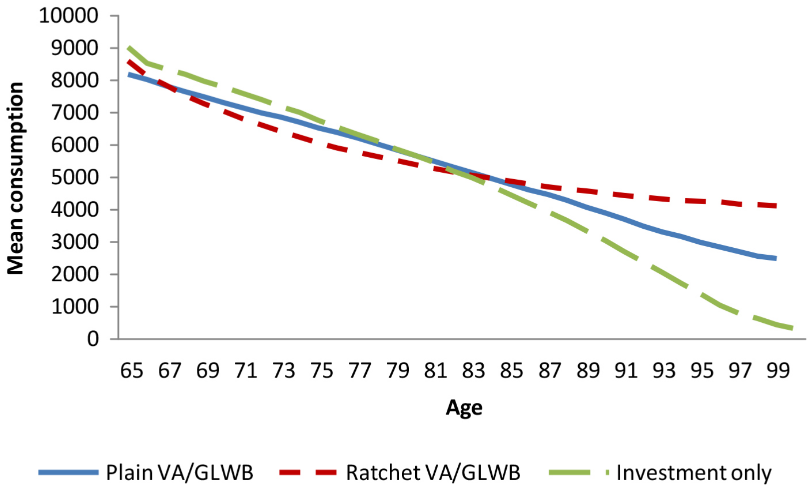
To determine how consumers might value the complex VA/GLWB product, we compute the value function by backward induction using the same grid points for the account value state space. Now, however, the value function is multivariate, depending on the retiree's account value as well as his guarantee value.[30] In the last period, he optimally consumes the greater of the account value or the guaranteed benefit from the VA/GLWB; this yields the multivariate value function for the last period. Next, we recompute the value function one period earlier using information from the final time point; the process is repeated until arriving at the first period. As before, we conduct 20,000 Monte-Carlo simulations for the asset process evolution as well as the mortality shocks. Information on optimal consumption patterns is obtained from the value function, and these 20,000 paths are used to determine expected utility and MWRs under the two different guarantee options. Next we explore results on optimal withdrawals for retirees who hold no VA, and then we compare withdrawal patterns for holders of a plain VA/GLWB and the ratchet VA/GLMB products.

Optimal Withdrawal Patterns. Figure 2 illustrates the average consumption path for all three portfolio alternatives, conditional on survival that period. The retiree without the annuity does consume at a higher rate during his first two decades, compared to the retiree with either a plain or a ratchet VA. The VA/GLWB product carries a penalty for excess withdrawals inasmuch as the retiree's guarantee base is reduced for the remaining lifetime; this constraint discourages early excess withdrawals.[31] Huang et al. (2014) show that policyholder should initiate withdrawals around age 60, as modeled here, but they also assume deterministic withdrawals and a risk-neutral perspective. Relaxing these two assumptions, we show that policyholders age 65 should withdraw more than the guaranteed rate, even though this implies less longevity protection in the future.

30 As before, grid points for the guarantee for the account value are set to $2,000 on average.

31 It can also catch up under the ratchet.

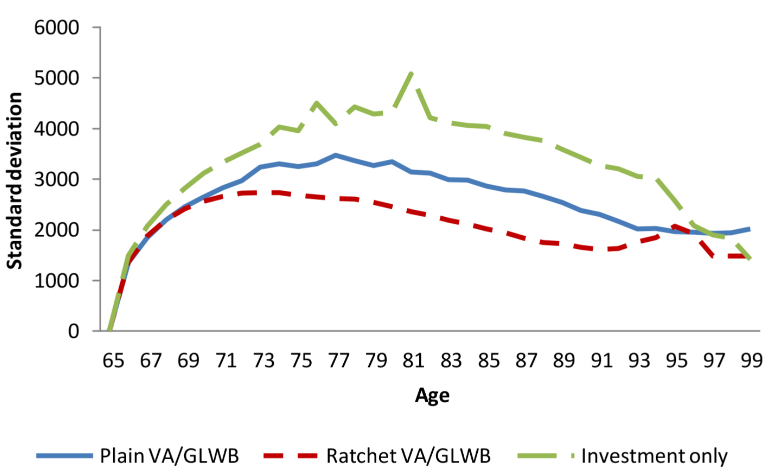
FIGURE 2: MEAN CONSUMPTION CONDITIONAL ON SURVIVAL UNDER THE THREE PORTFOLIO ALTERNATIVES



Note: This Figure displays the mean consumption path (above the Social Security benefit floor) for all individuals alive at each age. Three cases are considered: the plain and ratchet VA/GLWBs, and the investment -only case. Fees assumed for the plain VA/GLWB are 237 +75 bps and 237 + 100 bps for the ratchet VA/GLWB.

Figure 3 displays the standard deviation of consumption paths conditional on survival that period, again for all three portfolios. The retiree who lacks an annuity faces dramatically higher consumption volatility over the first two decades of retirement; this is clearly unappealing to the risk-averse. Thereafter, volatility declines significantly (from age 85) since people outlive their assets and must rely only on Social Security. Some do experience very high consumption due to very positive capital market returns, but in the end all survivors run out of assets. By contrast, the plain VA-holder has slightly lower consumption early on, but higher consumption later in life, compared to the non-annuitant. The retiree with the ratchet VA/GLWB consumes slightly less than his two counterparts until he attains his mid-80s, but thereafter his consumption is substantially higher than both of the other portfolios. This might seem surprising, as the guaranteed withdrawals under a ratchet VA/GLWB depend not only on earlier withdrawals but also on past investment performance, which could boost the volatility of the guarantee base and withdrawals. Yet the lower withdrawals early in retirement do not deplete the guarantee base as much as under the plain VA/GLWB and therefore provide more stability over the lifetime.

FIGURE 3: STANDARD DEVIATION OF CONSUMPTION CONDITIONAL ON SURVIVAL UNDER THREE PORTFOLIO ALTERNATIVES

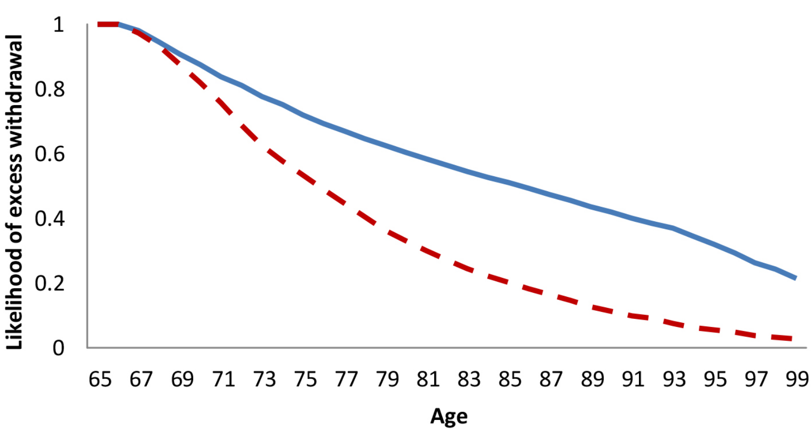


Note: This figure displays the standard deviation of the consumption path (above the Social Security benefit floor) for individuals alive. Three cases are considered: the plain and ratchet VA/GLWBs, and the investment-only case. Fees assumed for the plain VA/GLWB are 237 +75 bps and 237 + 100 bps for the ratchet VA/GLWB.

Figure 3 also shows that longevity protection in the later years does reduce consumption volatility compared to the investment-only retiree. Yet the standard deviation never falls below $1,500, implying that consumption at very old ages still varies importantly even in the case with a VA. This results from some people taking excess withdrawals and hence reducing their guarantee base.

Figure 4 illustrates the likelihood of making an excess withdrawal, i.e., withdrawing more than guaranteed withdrawal amount over time. For both types of VA buyers, the probability of taking out more than the guaranteed 5% withdrawal amount is very high, early in retirement. Thereafter, the likelihood of excess withdrawals falls with age. The plain VA/GLWB retiree has a probability of less than 40% at very old ages (i.e., after age 99), because he mostly uses the plain VA/GLWB as downside risk protection against longevity risk and bad market results. A ratchet VA/GLWB purchaser generally preserves more of his guarantee base and is hence less exposed to risk at very old ages.

FIGURE 4: LIKELIHOOD OF EXCESS WITHDRAWALS CONDITIONAL ON SURVIVAL: PLAIN VA/GLWB AND RATCHET VA/GLWB



Note: The figure displays the likelihood of making an excess withdrawal (i.e., withdrawing more than the guaranteed withdrawal amount), for individuals alive at each age. We represent these for both the plain and the ratchet VA/GLWBs. Fees assumed for the plain VA/GLWB are 237 +75 bps and 237 + 100 bps for the ratchet VA/GLWB.

Comparing Money's Worth Ratios (MWR) and Utility under each Portfolio. To value these portfolios more formally, we next calculate the Money's Worth Ratios and utility values as described above. For the MWR analysis, we assume that the remaining account value represents the death benefit and discount it with the risky return rate (since the remaining account value depends solely on the portfolio development and earlier withdrawals).[32] For an investor holding no VA, any remaining account value at death is treated as a bequest.

To make results more comparable across products, we use the anticipated portfolio return rates minus fees for the non-VA investment, setting to one the MWR for the pure investment product. Any divergence in the MWR from one in the case of the VA products indicates how the VAs' higher fees reduce net returns compared to not holding the annuity. Table 2 reports results. Here we see that the MWR for the plain VA/GLWB equals 0.92: this implies that for every dollar invested, a retiree could expect to receive 92 cents in benefits, *ex ante*. In other words, 8 cents per dollar are devoted to loads and the protections embedded in the plain VA/GLWB, compared to an equally risky non-annuity investment.[33] For the ratchet VA/GLWB, the MWR is 0.89, or three cents less per dollar of premium. This reduction is greater than the fee of 0.25% (25 bps), and it results from people optimally selecting different withdrawal paths over their lifetimes. Accordingly, the ratchet fee of 25 bps includes some security buffer due to the additional risk borne by the insurer.

32 It is worth noting that VAs frequently offer an enhanced death benefit which guarantees that when the retiree dies, his heirs will receive the greater of the current account value or the principal investment minus withdrawals. This enhanced death benefit is only offered until lifetime withdrawals are made, which usually commence after the second withdrawal from the contract. Hence withdrawing from the account only once does not trigger a reduction of the enhanced death benefit.

33 We treat the equity portfolio and the VA/GLWB investments as if they were equally risky, even though the included guarantee within the GLWB might have an impact on the overall portfolio exposure. But since we assume that both investments are in the same asset class, we use the same discount factors; this also makes it possible to explicitly determine the price for the included guarantee.

TABLE 2: MONEY'S WORTH RATIOS (MWR) AND ANNUITY EQUIVALENT WEALTH VALUES (AEW) FOR ALTERNATIVE PORTFOLIOS

	Money's Worth Ratios	Annuity Equivalent Wealth
Investment only (126 bps)	1.00	$100,000
Plain VA/GLWB (237 + 75 bps)	0.92	$111,064
Ratchet VA/GLWB (237 + 100bps)	0.89	$112,101
SPIA $6,950/year	0.81	$134,745
SPIA $7,950/year	0.93	$165,129

Note: VA/GLWB refers to variable annuity with a guaranteed withdrawal lifetime benefit; with or without ratchet refers to whether there is an annual step-up if the account value is sufficiently large; SPIA is a single premium immediate annuity. See text for further discussion.

To evaluate the insurance protection offered by the two VAs, Table 2 also reports Annuity Equivalent Wealth (AEW) values for a risk-averse consumer.[34] This AEW is a widely-used measure of how much non-annuitized wealth the individual lacking access to an annuity would be willing to pay to purchase the VA product; in other words, it is the utility equivalent wealth value making him indifferent between the two. As is clear from the table, both VA/GLWB products raise utility. The AEW of the plain VA/GLWB is roughly $111,000, meaning that having $100,000 in a plain VA/GLWB gives the same expected utility as $111,000 under the investment-only alternative. This 11% gain indicates that the longevity protection of the GLWB is worth more to the risk-averse retiree than the loss of 8 cents per dollar in the terms of the MWR. Having access to a ratchet VA/GLWB increases the AEW measure to around $112,000, for a 12% welfare gain. This slightly higher valuation is attributable to the lower consumption decline at older ages as well as the smaller consumption volatility over time. This comes at an additional cost of three cents per dollar over the plain VA. Since both the AEW measures and the premiums are rather similar to each other, we conclude that both products provide similar measures of protection for roughly similar prices.

For the sake of comparison, we also compute AEWs for a single-premium, immediate annuity (SPIA) with either a $6,950 or $7,950 monthly payment. The lower number is derived from market quotes indicating that a 65-year old male can receive a $6,950 yearly annuity payment. The higher number is obtained by multiplying the fair annuity pay-out[35] by the average MWR for annuitant mortality rates from Mitchell et al. (1999). Table 2 notes that the SPIA paying $6,950/month has a MWR of 81 cents per dollar invested (9 cents less than the ratchet VA/GLWB), but it boosts the AEW to around $135,000, or $24,000 more than the plain VA/GLWB and $23,000 more than the ratchet VA/GLWB. The SPIA paying $7,950 also has a higher MWR than the ratchet VA/GLWB; the AEW is around $165,000, or more than 40% over the ratchet VA/GLWB value. Accordingly, we conclude that the VA/GLWB combination does offer a higher expected utility than an investment-only alternative, but traditional SPIAs are probably more attractive than the typical VA/GLWBs examined here.

SENSITIVITY ANALYSES

To assess the robustness of our results, we also conduct sensitivity analyses with respect to three sets of parameter values: VA fees and guaranteed withdrawal rates, capital market assumptions, and mortality tables. An additional variant adds a bequest motive. Our base case is always the setup with annuitant mortality, the average VA fee of 237 bps, and $\mu = 0.0675$ and $\sigma = 0.18$.

Fees and guaranteed withdrawal rates. As noted above, some critics have cited high fees as a concern specific to variable annuities.[36] For this reason we investigate how sensitive consumer valuations of the VA/GLWB are to different

34 See Mitchell et al. (1999) for a full discussion of the AEW.

35 Using out life table, a fair annuity with zero transaction costs would pay a 65-year old male $8.618 per year for an initial $100,000 premium.

36 See for instance http://www.smartmoney.com/retirement/planning/whats-wrong-with-variable-annuities-9512/?zone=intromessage. The SEC also provides the following note of caution about VAs under their investor tips page: "Caution: You will pay for each benefit provided by your variable annuity. Be sure you understand the charges. Carefully consider whether you need the benefit. If you do, consider whether you can buy the benefit more cheaply as part of the variable annuity or separately (e.g., through a long-term care insurance policy), see http://www.sec.gov/investor/pubs/varannty.htm. "

charges. In particular, we ask what happens if an annuitant were to select either the most or the least expensive VA available in the market; accordingly, we decrease the account and investment fees to 129 bps (lowest-cost product found in the online market), or increase them to 280 bps (most expensive), leaving the GLWB rider fee untouched.[37]

Not surprisingly, higher fees reduce the MWRs as well as the expected utility of the plain VA/GLWB and the ratchet VA/GLWB (see Table 3). The difference in AEWs between the highest and lowest fee is in the 7% range, far less than the difference in AEWs between the base case and the life annuity. Changing fees also influences the drift of the Brownian motion μ, changing not only the VA/GLWB value, but also the optimal pattern of withdrawals. Our results do not depend on the reason for why the drift changes (i.e., whether the change in drift depends on different fee assumptions or a change in the capital market environment). Therefore, we discuss the qualitative impact of different fee assumptions on withdrawal behavior in the next section, where we explore how results differ given changes in capital market parameters.

TABLE 3: MONEY'S WORTH RATIOS (MWR) AND ANNUITY EQUIVALENT WEALTH VALUES (AEW) FOR ALTERNATIVE FEE ASSUMPTIONS

	Money's Worth Ratios	Annuity Equivalent Wealth
Plain VA/GLWB		
Base case (237 + 75bps)	0.92	$111,064
Low fee (129 + 75 bps)	0.99	$117,007
High fee (280 + 75 bps)	0.90	$109,071
Withdrawal % 4.5 (237 + 75 bps)	0.90	$105,524
Ratchet VA/GLWB		
Base case (237 +100 bps)	0.89	$112,101
Low fee (129 + 100 bps)	0.94	$118,039
High fee (280 +100 bps)	0.87	$111,744
Withdrawal % 4.5 (237 + 100 bps)	0.86	$103,656

Note: See Table 2.

In addition to fee variations, we also investigate how changes in the guaranteed withdrawal percentages might affect product valuations. In Table 1, we showed that most insurers offer a 5% withdrawal for a 65-year old retiree, but one competitor did have a 4.5% guaranteed withdrawal rate. If we reduce the guaranteed withdrawal to 4.5%, this has a relatively small impact on the MWR but a considerable impact on the AEW (see Table 3). The MWR decreases by roughly 1.5% and 3.7% for the plain and the ratchet VA/GLWB, respectively; this is comparable to the differences seen when the product had a 5% guaranteed withdrawal and the most expensive fee (280 bps). Meanwhile, the AEW decreases by more than 6.5% and 8.5%, respectively, compared to the standard fee of 237 bps, and around 10%-12% compared to the least expensive product. Retirees optimally react to the lower guaranteed withdrawal amounts by increasing their excess withdrawal probability, only slightly reducing the flows from the product. But a lower guaranteed withdrawal percentage implies less longevity protection, so the AEW is reduced -- not only by the reduction in MWR, but also by the lower longevity protection.

Capital market assumptions. Next we conduct sensitivity analysis with respect to capital market parameters: specifically, we vary the values of drift from 0.04 to 0.10, and from 0.10 to 0.25. Table 4 shows the resulting AEWs and MWRs. Here we see that the AEW is higher for the ratchet VA/GLWB compared to the plain product, for six of the nine scenarios. Only when we have very high returns and low volatility, or high returns and moderate volatility, or moderate returns and low volatility, is this relationship reversed. As volatility generally increases with returns, these are probably

37 This assumes that providers with higher/lower account fees also charge higher/lower GLWB and investment fees; otherwise we might find an even wider range of fees.

the least likely scenarios. Accordingly, a ratchet priced at 25 bps can be a valuable addition to a VA/GLWB from the consumer's perspective. Generally, the plain product has the highest MWR, except for the highest risk/return scenario, which is sensible since the ratchet becomes increasingly valuable with additional risk.

TABLE 4: MONEY'S WORTH RATIOS (MWR) AND ANNUITY EQUIVALENT WEALTH VALUES (AEW) FOR ALTERNATIVE CAPITAL MARKET SCENARIOS

	Plain VA/GLWB		Ratchet VA/GLWB	
	MWR	AEW	MWR	AEW
μ=0.04				
σ=0.10	0.85	$98,756	0.79	$98,373
σ=0.18	0.88	$100,484	0.83	$104,898
σ=0.25	0.91	$100,850	0.87	$109,915
μ=0.0675				
σ=0.10	0.90	$111,756	0.85	$105,983
σ=0.18	0.92	$111,064	0.89	$112,100
σ=0.25	0.95	$100,072	0.93	$116,870
μ=0.10				
σ=0.10	0.94	$135,488	0.90	$120,424
σ=0.18	0.95	$127,857	0.94	$122,658
σ=0.25	0.97	$121,703	0.99	$125,666

Note: See Table 2 for definitions. Fees assumed for the plain VA/GLWB are 237 +75 bps and 237 + 100 bps for the ratchet VA/GLWB.

From the insurer's perspective, the buyer's portfolio choice can have a substantial impact on the profitability of the VA/GLWB. Our highest return/volatility portfolio given the ratchet has a MWR slightly in excess of one; clearly this is not sustainable as it would imply a loss for the insurance company. At the same time, the product offers a higher expected utility for the policyholder than the base case scenario with $\mu = 0.0675$ and $\sigma = 0.18$. Accordingly, insurers will need to limit buyers' choices so they do not increase portfolio risk and return to the point that the guarantee becomes unprofitable for the insurer.[38] Our inspection of prospectuses for ratchet VA/GLWBs confirms that investment choices are, in fact, typically restricted.[39]

We also find that an increase in the drift μ for a given σ always increases the MWR; the AEW also rises as the expected portfolio return increases. A higher σ also boosts the MWR, since the GLWB provides downside risk protection. Individuals benefit more from upside risk in terms of average account values with higher volatility, while downside risk is to some extent buffered by the GLWB. This is particularly the case for the ratchet VA/GLWB as a higher volatility increases the probability of a significant step-up. Yet we also find that a higher σ for a given μ does not always increase the AEW and MWR. Without the ratchet, a higher at the lowest assumed drift is always beneficial: the chance to participate in upside risk is relatively appealing, compared to the return of the simpler VA portfolio. In the base case scenario with μ=0.0675, we see that the AEW is lower when is increased and slightly increases when is reduced compared to the base

38 Mahayni and Schneider (2012) show that a GMAB increases risk-taking during the accumulation period.

39 For instance the Allianz Connection Variable Annuity prospectus states that "If you select this benefit [the GLWB], we restrict your Investment options and rebalance your portfolio quarterly. [...]. These restrictions support the benefit's guarantee and [...] they may limit the upside potential." (Allianz 2012, p. 46). The Prudential Premier Advisor Variable Annuity prospectus states that "Each living benefit requires your participation in a predetermined mathematical formula that may transfer your account value between the Sub-accounts you have chosen from among those we permit with the benefit (i.e., the "permitted Sub-accounts") and certain bond portfolio Sub-accounts of AST. [...] Although not guaranteed, the optional living benefit investment requirements and the applicable formula are designed to reduce the difference between your Account Value and our liability under the benefit. Minimizing such difference generally benefits us by decreasing the risk that we will use our own assets to make benefit payments to you. Though the investment requirements and formulas are designed to reduce risk, they do not guarantee any appreciation of your Account Value. In fact, they could mean that you miss appreciation opportunities in other investment options." (Prudential 2012, p. 47)

case value of . In other words, for the given μ=0.0675, the optimal portfolio from an investor's point of view would be a less volatile. Yet the question remains whether it is possible to reduce the riskiness of the assumed portfolio without impacting returns. For the highest drift, we assume (μ=0.10) higher volatility increases the MWR but decreases the AEW. This indicates that downside risk protection no longer outweighs the gain due to high returns, given relatively high returns in the VA portfolio.

For the ratchet VA/GLWB, increasing the volatility for a given drift increases both the MWR and the AEW for all of our scenarios. But differences in AEW become smaller when the volatility is high. Accordingly, we suppose that there will be a finite optimal volatility for any given μ which is just greater than σ=0.25. Again, this indicates that the ratchet VA/GLWB leads to riskier portfolios. Nonetheless, the combination of the highest return and volatility, i.e. μ=0.10 and σ=0.25, gives a higher AEW value than in the base case, indicating that investors should increase portfolio risk and return compared to base case parameters. Differences in AEW among the different portfolios are much greater than observed differences for fee variations. The best VA/GLWB portfolio in terms of AEW (plain VA/GLWB with return and lowest risk) fares a little better than the more expensive SPIA. Nevertheless, obtaining a portfolio offering a 10% return and 10% standard deviation is unlikely at present.

Different life tables. Next we compute the impact of using the population SSN 2006 life table instead of the annuitant table. To provide a sense of how these tables differ, the remaining life expectancy of a male age 65 is 20.5 years under the annuitant table, while it is 17.6 years for the SSN table. Not surprisingly, using the population tables does change the MWR of both VA products of interest here: SSN mortality reduces the MWR ratio (with or without the ratchet) by roughly 1-3 cents per dollar. These differences are less pronounced than in conventional fixed annuities, for which Mitchell et al. (1999) reported differences of roughly 10 cents. This is because for the VAs examined here, retirees can adjust their withdrawals in light of their lower life expectancy. Moreover, retirees who survive end up relying on the GLWB in their later years, so differences in life expectancy do not matter as much as for traditional fixed annuities.

Bequests. We also investigate how a simple bequest motive in the utility model might impact the appeal of the base case plain and ratchet VA/GLWBs, versus the investment-only alternative and traditional annuities. When retirees care about leaving a bequest, this will influence their consumption/withdrawal patterns when alive. We follow Lockwood (2013) in modeling the bequest motive (who generalizes DeNardi, 2004, and Ameriks et al. 2011). The utility of bequest is expressed as follows:

$$u_{bequest} = \left(\frac{b}{1-b}\right)^{RRA} \cdot \frac{\left(\frac{b}{1-b} c_{bequest} + B\right)^{1-RRA}}{1-RRA} ,$$

where $b \in (0,1)$ denotes the strength of the bequest motive, and $c_{bequest}$ denotes the minimum consumption threshold an individual needs before caring about a bequest. B denotes the bequest. For parameterization we use Lockwood's (2013) estimates of b=0.47 and $c_{bequest}$=20,416 estimated from individuals over age 65 in the Health and Retirement Study.

As shown in Table 5, adding the bequest motive slightly decreases withdrawals and the probability of making an excess withdrawal, but it does not change our results substantively. There are also few differences in the MWRs. Generally speaking, the AEW given a bequest motive is lower, due to the fact that the embedded withdrawal guarantee cannot be bequeathed. For the plain VA/GLWB, the AEW decreases from $111,064 without a bequest to $107,484 under a bequest motive, for the ratchet VA/GLWB from $112,101 to $108,762.

TABLE 5: MONEY'S WORTH RATIOS AND ANNUITY EQUIVALENT WEALTH VALUES FOR ALTERNATIVE PORTFOLIOS AND ASSUMING A BEQUEST MOTIVE

	Money's Worth Ratios	Annuity Equivalent Wealth
Investment only (126 bps)	1.00	$100,000
Plain VA/GLWB (237 + 75 bps)	0.92	$107,484
Ratchet VA/GLWB (237 + 100 bps)	0.89	$108,762
SPIA 6,950$	0.81	$130,365
SPIA 7,950$	0.93	$151,365

Note: See Table 2 for definitions.

The AEW of a traditional SPIA providing the lower payment falls from $134,745 without a bequest to $130,065 with a bequest. For the higher payout SPIA, the AEW is reduced much more ($165,129 without a bequest vs. $151,365 with): this is because the individual now faces a direct tradeoff between his own consumption versus leaving a bequest.

CONCLUSIONS

Variable annuities with guaranteed minimum lifetime withdrawal benefits (VA/GLWBs) are important retirement payout products, as they offer retirees access to the capital market while providing longevity protection and a flexible lifetime income stream. We show how a risk-averse retiree who holds his retirement assets in a VA/GLWB can optimally utilize this product with or without ratchets. We conduct a market analysis to model our stylized VA/GLWB product, along with sensitivity analyses to illustrate a variety of products available. In view of the many products available in the marketplace, their wide price differences, and their different guarantees and portfolio choices, our approach should be seen as illustrative and our results appropriate for an 'average' product. Financial advisers and firms seeking to provide consumer guidance may wish to automate this sort of calculation for specific policies.

We show that the utility-maximizing withdrawal patterns from the VA/GLWB are driven by a tradeoff between financial returns and maintaining longevity protection. We then compare these withdrawal patterns to what retirees can do if they invested in either a simple investment portfolio or a traditional SPIA annuity. We also calculate Money's Worth Ratios taking into account that usage of the embedded options heavily depends on the consumer's preferences. Our calculations depart from a conventional risk-neutral valuation approach, as the embedded flexibility in the product enables consumers to tailor their payout stream according to their preferences. Accordingly, obtained values are preference dependent and not unique as would be required for a risk-neutral valuation approach. Our MWRs for the plain VA/GLWB are around 0.92, implying that the buyer pays 8 cents per $1 premium for the protections embedded in this product. Including a ratchet produces a MWR of 0.89, or 3 cents less per dollar of premium.[40] The MWR is lower for the latter product since people optimally select different withdrawal paths over their lifetimes. We also show the impact of changes in the assumed portfolio which imply MWRs between 0.79 and 0.99.

Our MWR results differ from those reported by studies undertaken prior to the financial crisis, which suggested that GLWBs were underpriced (and therefore would have a MWR greater than one). In fact, those products provided much more generous benefits, whereas our work focuses on products available post-financial crisis. Moreover, we do not price the GLWB independently of the VA but instead we look at the compound product, since GLWBs are usually only accessible through the purchase of an accompanying VA. This allows potential cross-subsidization between the product components. Our MWR evaluations do not start from a risk-neutral option value optimization framework; instead we posit a risk-averse retiree seeking longevity protection and willing to forgo some money in expectation. These factors taken together show

40 These figures are comparable to simpler MWRs for SPIA annuities using Social Security mortality tables. See also Mitchell et al. (1999) and James and Song (2001).

that a VA/GLWB combination can be a sustainable contract from the insurer's perspective as long as portfolio risk is not too high; this justifies the ongoing availability of these products over the last several years.

We use the Annuity Equivalent Wealth (AEW) ratio to compare utility from the VA/GLWB to a fixed annuity and a mutual fund investment. For a plain VA/GLWB, the AEW is around 11% higher than an investment portfolio with the same asset allocation, while the ratchet product increases the measure to 12%. If the returns and volatility are smaller, holding a VA/GLWB product becomes less attractive, whereas a higher return and volatility can increase the AEW by roughly 25%. Of course, any given VA/GLWB will differ in some way from what we model here in terms of specific features and pricing, but our goal is to demonstrate an approach to valuation rather than to highlight any particular product.

Since the AEW measures and the premiums are relatively similar, we also conclude that both products offer measures of protection for roughly similar prices; by contrast, traditional single premium immediate annuities are more attractive than the typical VA/GLWBs examined here. Sensitivity analysis does not overturn these conclusions, though we do find that permitting riskier portfolios inside the VAs – especially the ratchet product – can have solvency implications for the product provider. Particularly in the wake of the financial crisis, some VA providers have adopted "volatility management strategies," with one firm recently dropping 26 investment options from its VA investment lineup.[41] Yet it seems clear that annuity products are likely to grow in popularity as Baby Boomers move into retirement, due to their potential to provide flexible yet guaranteed minimum income streams in retirement.[42]

Traditional defined benefit pension plans have given way to defined contribution plans in the United States over the past half century, devolving to retirees the responsibility for handling their own retirement payouts. Our work shows that VA/GLWBs can serve as a useful substitute, depending on pricing and individual risk preferences.

41 See for instance Mercado (2013).

42 A recent survey concluded that 86% of non-retired Americans sought guaranteed income streams in retirement (Marketwatch.com, 2011).

REFERENCES

Ameriks, J., Caplin, A., Laufer, S. and Van Nieuwerburgh, S. (2011): "The Joy of Giving or Assisted Living? Using Strategic Surveys to Separate Public Care Aversion from Bequest Motives." *Journal of Finance*, Vol. 66, pp. 519–561.

Bacinello, A. R., Millossovich, P., Olivieri, A., and Pitacco, E. (2011). "Variable Annuities: A Unifying Valuation Approach." *Insurance: Mathematics and Economics*, *49*(3), 285-297.

Bauer, D., J. Russ and A. Kling (2008): "A Universal Pricing Framework for Guaranteed Minimum Benefits in Variable Annuities." *Astin Bulletin*, Vol. 38(2), pp. 621-651.

Bernard, C., A., Mackay, and M. Muehlbeyer. (2014). "Optimal Surrender Annuity Guarantees." *Insurance: Mathematics and Economics*. 55: 116-128.

Blanchet, D. M. (2011): "The Expected Value of a Guaranteed Minimum Withdrawal Benefit (GMWB) Annuity Rider." *Journal of Financial Planning*, July, pp. 52-61

Blanchet, D. M. (2012): "The Optimal Portfolio Allocations with GMWB Annuities." *Journal of Financial Planning*, October, pp. 46-56.

Brown, J. R., A. Kapteyn, E. Luttmer, and O. S. Mitchell. (2013). "Do Consumers Know How to Value Annuities? Complexity as a Barrier to Annuitization." NBER Working Paper No. 19168.

Brown, J.R. and J. M. Poterba (2006): "Household Ownership of Variable Annuities." *Tax Policy*. Vol. 20, pp. 163-191.

Chai, J., W. Horneff, R. Maurer, and O.S. Mitchell (2011): "Optimal Portfolio Choice over the Life Cycle with Flexible Work, Endogenous Retirement, and Lifetime Payouts." *Review of Finance*. Vol. 15(4), pp. 875-907.

Charupat, N. and M. A. Milvesky (2002): "Optimal Asset Allocation in Life Annuities – A Note." *Insurance: Mathematics and Economics*. Vol. 30(2), pp. 199-209.

Chen, Z., K., Vetzal, and P.A. Forsyth. (2008): "The Effect of Modelling Parameters on the Value of GMWB Guarantees." *Insurance: Mathematics and Economics*, *43*(1), pp. 165-173.

Dai, M., Y. Kuen Kwok and J. Zong (2008): "Guaranteed Minimum Withdrawal Benefits in Variable Annuities." *Mathematical Finance*. Vol. 18(4), pp. 595-611.

Davidoff, T. (2009): "Housing, Health and Annuities." *Journal of Risk & Insurance*, Vol. 76 (1), pp. 31–52.

DeNardi, M. (2004): "Wealth Inequality and Intergenerational Links." *Review of Economic Studies*, Vol. 71(3), pp. 74-768.

Doyle, S., O.S. Mitchell, and J. Piggott (2004): "Annuity Values in Defined Contribution Retirement Systems: Australia and Singapore Compared." *Australian Economic Review*, Vol. 37(4), pp. 402-416.

Feng, R., and H.W. Volkmer. (2012), "Analytical Calculation of Risk Measures for Variable Annuity Guaranteed Benefits." *Insurance: Mathematics and Economics*, *51*(3), pp. 636-648.

Fong, Wai M. (2002): "On the Cost of Adverse Selection in Individual Annuity Markets: Evidence from Singapore." *Journal of Risk and Insurance*. Vol. 69(2), pp. 193-207.

Fong, J.H.Y., O.S. Mitchell, and B. S. K. Koh. (2011): "Longevity Risk Management in Singapore's National Pension System." *Journal of Risk and Insurance*. Vol. 78(4), pp. 961-984.

Forsyth, P. and K. Vetzal. (2012): "A Methodology for the Valuation of Guaranteed Lifelong Withdrawal Benefits." Working paper available at https://cs.uwaterloo.ca/-paforsyt/glwb.pdf.

Gao, J. and E R. Ulm (2012). "Optimal Consumption and Allocation in Variable Annuities with Guaranteed Minimum Death Benefits." *Insurance: Mathematics and Economics*, Vol. 51(3), pp. 586-598.

Gomes, F. and A. Michaelides (2005): "Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence." *Journal of Finance*, Vol. 60(2), pp. 869-904.

Holz, D., J. Russ and A. Kling (2008): "GMWB for Life - An Analysis of Lifelong Withdrawal Guarantees." *Zeitschrift fuer die gesamte Versicherungswissenschaft,* Vol. 101, pp. 305-325.

Horneff, W., R. Maurer, O.S. Mitchell, and I. Dus (2007): "Following the Rules: Integrating Asset Allocation and Annuitization in Retirement Portfolios." *Insurance: Mathematics and Economics*. Vol. 42, pp. 396-408.

Horneff, W., R. Maurer, O.S. Mitchell, and M. Stamos (2009): "Asset Allocation and Location over the Life Cycle with Survival-Contingent Payouts." *Journal of Banking and Finance*. Vol. 33, pp.1688-1699.

Horneff, W., R. Maurer and R. Rogalla (2010a): "**Dynamic Portfolio Choice with Deferred Annuities.**" *Journal of Banking and Finance***. Vol. 34, pp.** 2652-2664.

Horneff, W., R. Maurer, O.S. Mitchell, and M.Z. Stamos (2010b): "Variable Payout Annuities and Dynamic Portfolio Choice in Retirement." *Journal of Pension Economics and Finance*. Vol. 9, pp. 163-183.

Huang, H. M.A. Milevsky, and T.S. Salisbury. (2014). "Optimal Initiation of a GLWB in a Variable Annuity: No Arbitrage Approach." *Insurance: Mathematics & Economics*. 56(May): 102-111.

Insured Retirement Institute. (IRI, 2012). *Variable Annuities Net Assets Reach All-time High*. http://www.irionline.org/news/article/id/647. Downloaded 06/08/2012.

Insured Retirement Institute. (IRI, 2010): *2010 Annuity Factbook*. Washington, DC: IRI.

Kling, A., F. Ruez and J. Russ (2011): "The Impact of Stochastic Volatility on Pricing, Hedging, and Hedge Efficiency of Variable Annuity Guarantees." *Astin Bulletin*, Vol. 41, pp.511-545.

James, E. and X. Song (2001): "Annuity Markets Around the World: Money's Worth and Risk Intermediation." SSRN: http://ssrn.com/abstract=287375

Lockwood, L. (2013): "Bequest Motives and the Decision to Self-Insure Late-Life Risks." *Review of Economic Dynamics*. Vol 15(2), pp 226-243.

LIMRA (2010): *Variable Annuity Guaranteed Living Benefit Election Tracking Survey Report*. http://www.limra.com/newscenter/NewsArchive/ArchiveDetails.aspx?prid=117 Downloaded 01/06/2011

Mahayni, A. and J. Schneider (2012): "Variable Annuities and the Option to Seek Risk: Why Should They Diversify?" *Journal of Banking and Finance*, Vol. 36, pp. 2417-2428

Marketwatch.com (2011): "Guaranteed Income in Retirement Trumps Having 401(k) Plan." http://www.marketwatch.com/story/guaranteed-income-in-retirement-trumps-having-401k-plan-2011-10-18. 11/18/11.

Maurer, R., O.S. Mitchell and R. Rogalla (2010): "The Effect of Uncertain Labor Income and Social Security on Lifecycle Portfolios." In R. Clark and O.S. Mitchell, eds. *Reorienting Retirement Risk Management*. Oxford: OUP, pp. 107-121.

Maurer, R., O. S. Mitchell, R. Rogalla., and V. Kartashov (2013): "Lifecycle Portfolio Choice with Stochastic and Systematic Longevity Risk, and Variable Investment-Linked Deferred Annuities." *Journal of Risk and Insurance.* 80(3): pp. 649–676.

Mercado, D. (2013): "AXA Latest Insurer to Yank Generous VA Options." Investmentnews.com. at http://www.investmentnews.com/article/20130401/FREE/130409996&utm_source=indaily-20130401&utm_medium=in-newsletter&utm_campaign=investmentnews&utm_term=text. Downloaded 4/4/13.

Milevsky, M.A. and S.E. Posner (2001): "The Titanic Option: Valuation of the Guaranteed Minimum Death Benefit in Variable Annuities and Mutual Funds." *Journal of Risk and Insurance*. Vol.68(1), pp. 93-128

Milevsky, M.A. and T. S. Salisbury (2008): "Financial Valuation of Guaranteed Minimum Withdrawal Benefits." *Insurance: Mathematics and Economics*. Vol. 38(1), pp. 21-38.

Mitchell, O.S., J.M. Poterba, M. J. Warshawsky and J.R. Brown (1999): "New Evidence on the Money's Worth of Individual Annuities." *American Economic Review*. Vol. 89(5), pp. 1299-1318.

Moenig T. and D. Bauer. (2012): "Revisiting the Risk-Neutral Approach to Optimal Policyholder Behavior: A Study of Withdrawal Guarantees in Variable Annuities.". Working Paper. http://www.uni-ulm.de/fileadmin/website_uni_ulm/mawi.inst.140/Vortr%C3%A4ge/2011_MoenigBauer_RNVtoPHbehavior.pdf

Orman, S. nd. "Annuities." www.suzeorman.com/igsbase/igstemplate.cfm?SRC=MD012&SRCN=aoedetails&GnavID=84&SnavID=29&TnavID=&AreasofExpertiseID=107

Pfau, W. (2011): "GLWBs: Retiree Protection or Money Illusion? Advisor Perspective." December 13. http://advisorperspectives.com/newsletters11/GLWBs-Retiree_Protection_or_Money_Illusion.php

Piscopo, G. and S. Haberman. (2011). "The Valuation of Guaranteed Lifelong Withdrawal Benefit Options in Variable Annuity Contracts and the Impact of Mortality Risk." *North American Actuarial Journal,* Vol. 11, pp. 59-76

Poterba J., S. Venti and D. Wise (2012): "Were They Prepared for Retirement? Financial Status at Advanced Ages at the HRS and AHEAD Cohorts." *NBER Working Paper No. 17824*

Prudential (2012): *New York State Prospectus of the Premier Advisor Variable Annuity*. http://www.prudential.com/media/managed/documents/pruannuities_investor/ppr_ny_pros.pdf?siteID=25. Downloaded 06/08/2012.

Shah, P. and D. Bertsimas (2008). "An Analysis of the Guaranteed Withdrawal Benefits for Life Option." *Available at SSRN 1312727*.

Smartmoney.com (2011): "What's Wrong with Variable Annuities." August. http://www.smartmoney.com/retirement/planning/whats-wrong-with-variable-annuities-9512/

Social Security Administration (SSA, 2011): *Monthly Statistical Snapshot*. February www.ssa.gov/policy/docs/quickfacts/stat_snapshot/

Society of Actuaries (SOA, 2005). *RECORD*. Volume 31, No. 1, 2005. http://www.google.com/url?sa=t&rct=j&q=&esrc=s&frm=1&source=web&cd=1&ved=0CCQQFjAA&url=http%3A%2F%2Fwww.soa.org%2Flibrary%2Fproceedings%2Frecord-of-the-society-of-actuaries%2F2000-09%2F2005%2Fmay%2Frsa05v31n13pd.pdf&ei=cwsJU9VB4snRAcSqgOAL&usg=AFQjCNH0ty0XClh1ldTswHFQPUIf2q7YiA&bvm=bv.61725948,d.dmQ

Society of Actuaries (SOA, 2006): *Annuity 2000 Mortality Table*. Available at http://soa.org

Ulm, E. (2006): "The Effect of the Real Option to Transfer on the Value of Guaranteed Minimum Death Benefits." *Journal of Risk and Insurance*. Vol. 73(1), pp. 43-69

Ulm, E. (2010): "The Effect of Policyholder Transfer Behavior on the Value of Guaranteed Minimum Death Benefits in Annuities." North American Actuarial Journal. Vol. 14(1), pp. 16-37

Viceira, L. (2001): "Optimal Portfolio Choice for Long-horizon Investors with Nontradeable Labor Income." *Journal of Finance*. Vol. 55, pp. 1163–1198.

Yaari, M. (1965): "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer." *Review of Economic Studies*. Vol. 32(2), pp. 137–50.

Yang, S. S., and Dai, T. S. (2013): A Flexible Tree for Evaluating Guaranteed Minimum Withdrawal Benefits under Deferred Life Annuity Contracts with Various Provisions." *Insurance: Mathematics and Economics*, *52*(2), pp. 231-242.