HONG KONG MONETARY AUTHORITY

Research Memorandum 12/2017

11 August 2017

WHAT'S KEEPING US LONG-TERM INTEREST RATES SO LOW?

Key Points:

- *Long term US Treasury (UST) yields, which fell drastically in the aftermath of the global financial crisis (GFC), have stayed low despite the repeated rate hikes by the Federal Reserve since 2015.*

- *To study the driving forces behind the compressed US Treasury yields, we use a dynamic term structure model to extract a demand factor that is embedded in the UST yield curve and examine how this factor contributes to yield compression. The demand factor, which resembles a convenience yield, captures the information related to the demand for UST due to their safety and liquidity attributes.*

- *Our estimation suggests that the demand factor was underscored by a number of market forces during the Greenspan Conundrum (2004 –2006) and the post-GFC period:*
  - *Foreign purchases, safe asset demand and search for yield were key forces keeping UST yields low during both episodes.*
  - *QE1, QE2 and OT (but not QE3) also contributed to suppressing yields in the post GFC period.*

- *The Fed's balance sheet normalisation is expected to cause long term interest rates to rise through its impact on the demand factor, subject to uncertainties about the US macroeconomic performance as well as the demand from major non-Fed UST investors in the coming years.*

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I. INTRODUCTION

The unconventional monetary policies adopted by major central banks since the global financial crisis have manifested into a drastic decline in long-term interest rates globally. While the Federal Reserve (Fed) has started to raise the target range of the federal funds rate (FFR) since December 2015, the US long-term interest rates remain low which result in a flattening of the US Treasury (UST) yield curve.[1] Against this background, this paper examines the driving forces behind compressed UST yields and analyses how the yields may behave as the Fed normalises monetary policy.

Based on a dynamic term structure model, we extract a demand factor embedded in the UST yield curve and examine to what extent this demand factor has contributed to the yield compression. Specifically, we decompose the yield curve into three major components: (1) the long-term mean (similar to the idea of the natural interest rate, which is in line with the long-term potential growth of the economy); (2) the short rate (which is in line with the FFR); and (3) the demand factor. In the term structure model the demand factor captures the information related to the demand for UST due to their safety and liquidity attributes not captured by the natural rate or the short rate. The demand factor resembles the convenience yield in that investors are willing to forgo certain interest in exchange for holding a highly liquid and safe debt instrument. The model assumes that the sum of the short rate and the demand factor is the effective discount factor used by investors in pricing bonds. As a result, strong demand for the UST will bolster the demand factor, hence leading to a decline in the model bond yield.

The existence of convenience yield in the UST has long been documented in previous studies. Krishnamurthy and Vissing-Jorgensen (2012) argue that the common practice of using the short-term Treasury yields as the asset pricing models' risk-free interest rate is incorrect. The observed Treasury interest rate is lower than the "true" risk-free rate. A number of studies have

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[1] The Federal Reserve Open Market Committee (FOMC) has raised its policy rate four times since December 2015. As of June 2017, the FFR has since increased by nearly one percentage point but the 10-year Treasury yields has remained almost unchanged. This divergence in short-term and long-term interest rates movements appears reminiscent of the Greenspan conundrum when long-term Treasury yields declined amid rate hikes by the FOMC from 2004 to 2005. See Backus and Wright (2007) for a detailed study for the Greenspan conundrum.

quantified convenience yield and found it to be a significant component of equilibrium bond prices (e.g., Krishnamurthy (2002), Longstaff (2004), Fontaine and Garcia (2012), Krishnamurthy and Vissing-Jorgensen (2012), Smith (2012), Greenwood and Vayanos (2014), and Valchev (2016)). The average (annualized) convenience yield on UST ranges from 75 to 166 basis points (bps), and the estimates of the standard deviation range from 45 to 115 bps.

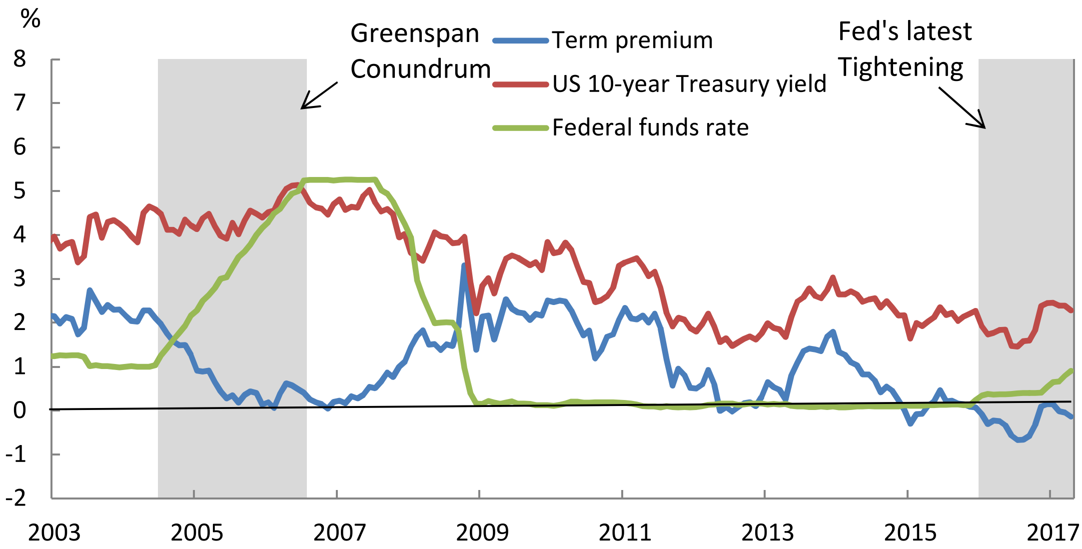
Typically, convenience yield identified in previous studies is a point estimate that summarises the average behaviour of convenience yield over a certain period. In contrast, the demand factor identified by our model is available in weekly frequency, which allows us to conduct statistical inference with other exogenous variables that may be related to the demand for UST. The most relevant exogenous variable prior to the global financial crisis (GFC) was the reserve accumulation of Asian and oil-producing economies that might have compressed long-term UST yields during the Greenspan conundrum. Since the outbreak of the GFC, the demand for UST was likely to be enhanced by the Fed's unconventional monetary policies. In particular, three rounds of quantitative easing and two rounds of operation twist were both targeted at purchasing long-term UST to lower long-term borrowing costs of the economy. Based on a regression model with the demand factor as the endogenous variable, we find that the foreign holding of UST, as well as the dummy variables for the months of executing the quantitative easing and operation twist, are important drivers for UST demand. Given these empirical results, the Fed's plan (Federal Reserve, 2017) to normalise its balance sheet by disinvesting its holding of UST securities is likely to *ceteris paribus* reverse the effects caused by unconventional monetary policies on UST yields.

This paper is organised as follows. The next section discusses some stylised facts on the 10-year UST. Section 3 describes the term structure model of bond yields. Section 4 shows the data used in this study and the empirical models for explaining the behaviour of the demand factor. The final section concludes.

II. RECENT PERFORMANCE OF THE LONG-TERM UST YIELDS

In discussing long-term interest rates, it is useful to note that long-term rates can be decomposed into two components: (1) the expected future short-term rates; and (2) the term premium. Intuitively, the term premium is the additional return demanded by investors to hold a long-term bond as opposed to rolling over a series of shorter-term bonds over the same period.

Figure 1. US 10-year Treasury yield, term premium and the federal funds rate



Sources: Federal Reserve Bank of New York and Bloomberg

Figure 1 graphs the FFR, 10-year UST yields and its term premium component. The 10-year yield and term premium generally move in tandem with each other, with two notable exceptions. The first episode occurred during the Greenspan conundrum in 2004-2006 when the Fed repeatedly raised interest rates and yet 10-year UST yield remained stable because of the significant fall in the term premium. The second episode occurred in the onset of the 2008 global financial crisis when the Fed aggressively cut interest rates to zero and yet the term premium increased notably, resulting in a smaller-than-expected drop in the 10-year yield. The seemingly disconnect between the 10-year yield and the term premium is probably due to the countercyclical nature of the latter: highs during recessions and lows during expansions.[2] The term premium remains at a very low level currently, which also happens to be a time when economic slack in the US is diminishing. Although it remains to be seen whether the low UST term premium is due to structural or cyclical factors, economists have offered three explanations that could affect the term premium.[3]

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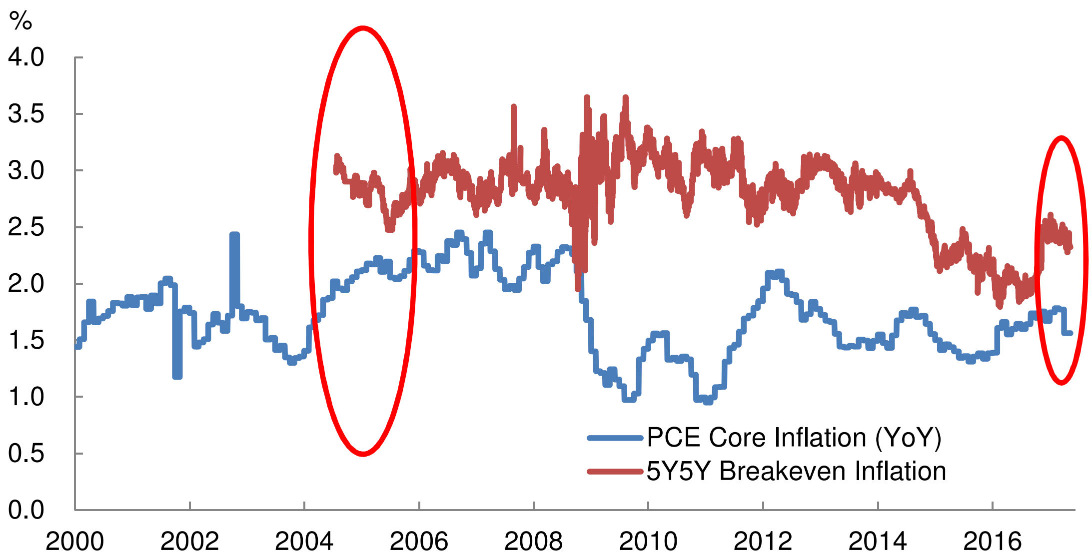
[2] See Chung et al. (2017).

[3] These explanations are related to several recent studies. The secular stagnation hypothesis put forward by Hansen (1939), which explained low long-term interest rates, is revitalized by Summers (2014) and Eggertsson et al. (2017). They argue that the prolonged low interest rate environment is caused by weak investment demand and excess saving. In a comprehensive survey, Rachel and Smith (2015) argue the fall in the relative price of capital and low public investment are the cause of weak investment, but the increase in saving is due more to population aging, higher inequality and precautionary saving.

First, low inflation or even deflation became the major concern of market participants amid accommodative monetary policies outside the US (Campbell et al., 2016). This is similar to the Greenspan conundrum, where declines in long-term inflation expectation weighed on the term premium (Figure 2).[4]

Figure 2. US long-term inflation expectation

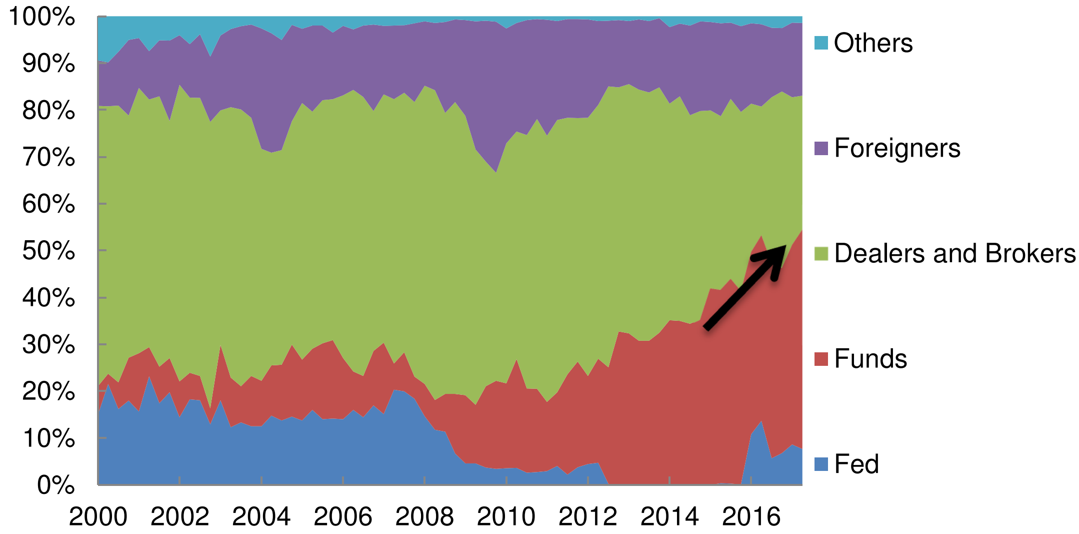


Source: Bloomberg

Second, in the primary market, increased auction participation by the Fed and investment funds, most of which tend to hold these bonds till maturity, has reduced the supply of long-term bonds available to the market (Figure 3). According to Bauer and Rudebusch (2014), reduced supply of long-term bonds tends to lower the term premium.

4 The long-term inflation expectation is measured by the 5-year, 5-year inflation swap rate. Specifically, it is calculated based on the difference between the 10-year and 5-year inflation swap rates.

Figure 3. Share of long-term US Treasuries auction by investor categories



Note: Fed refers to Federal Reserve Banks System Open Market Account. Funds refer to mutual funds, money market funds, hedge funds, money managers, investment advisors, pension and retirement funds, and insurance companies.

Source: US Department of the Treasury and staff estimates.

Third, in the secondary market, foreign investors continue to treat UST as a safe haven and flight-to-safety demand can drive UST yields significantly lower in event of geopolitical risks. For the Greenspan Conundrum, it is often argued that global excess saving (caused by emerging market economies running large current account surpluses) compressed UST yields. The recent decline in the term premium is attributable to the abundant liquidity from accommodative monetary policies by foreign central banks (Bauer and Rudebusch, 2016). Indeed, the New York Fed's Survey of Primary Dealers in July 2016 cited "spillovers from low/declining yields abroad" as the most important factor for declines in long-term UST yields in early 2016.

III. THE TERM STRUCTURE MODEL[5]

We assume a state vector $X_t = (r_t, \theta_t, L_t)$, where r is the short rate factor, θ the long-term mean factor and L the demand factor. All variables are unobservable but can be inferred from observed yields through the bond-pricing model. The three factors are assumed to be uncorrelated to each other. Specifically, the short rate is described by the following process:

$$dr_t = \kappa(\theta_t - r_t)dt + \sigma\sqrt{r_t}dZ_r \qquad (1)$$

where κ determines the speed of the mean-reverting drift towards the long-term mean θ_t, σ is the volatility and dZ_r is a standard Brownian motion. The long-term mean θ of the short rate in turn follows:

$$d\theta_t = \alpha(\beta - \theta_t)dt + \eta\sqrt{\theta_t}dZ_\theta \qquad (2)$$

where α, β and η are the mean-reversion parameters, long-term mean, and volatility of the Brownian motion respectively. The specification in Eqs.(1) and (2) follows Balduzzi et al. (1998) and are sometimes referred to as the stochastic mean model in the literature. Finally, the stochastic process for the demand factor L_t is:

$$dL_t = -\xi L_t dt + \gamma dZ_L \qquad (3)$$

where ξ and γ are the drift term and volatility parameter of the Brownian motion Z_L. L_t proxies for macro and market information the market participants care about when trading the bonds. Such information is not contained in the other state variables. A similar factor is used by Piazzesi (2005), who assumes an exogenous process to capture other information not contained in the other state variables that could affect the yield curve. Given that investors value the safety and liquidity of UST, they are willing to forego an amount of interest in exchange for owning a highly liquid and safe debt instrument. Such interest is a significant time-varying convenience-yield component in UST yields, i.e., pushing down the yields, and is captured by L_t in the model. A larger convenience yield reflects higher demand for UST.

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[5] This section draws freely from Hui et al. (2017).

Given Eqs. (1)-(3), the price of a zero-coupon bond with a maturity at time $\tau = T -$ is given by:

$$P_t(\tau, r, \theta, L) = E_t\left[exp\left(-\int_t^T (r_t + L_t) \quad \right)\right] \tag{4}$$

where E_t the expectation operator.[6] We assume the expectation is taken by the market in such a way that the adjusted discount rate $(r_t + L_t)$ is the effective interest rate matching the observed bond yields. L_t is therefore introduced as a reduced form fashion to capture the demand for UST that carries a convenience yield. Krishnamurthy and Vissing-Jorgensen (2012) argue the observed Treasury interest rate is lower than the "true" risk-free interest rate by an amount of the convenience yield. The construction of the term structure model suggests that the short rate r_t contains the information on the "true" risk-free interest rate. Strong demand for UST will cause negative L, that pushes the effective interest rate $(r_t + L_t)$ down.

Duffie and Kan (1996) shows that Eq. (4) has the solution of the form:

$$P_t(\tau, r, \theta, L) = exp[A(\tau) - B(\tau)r_t - C(\tau)\theta_t - D(\tau)L_t] \tag{5}$$

where $A(\tau), B(\tau), C(\tau), D(\tau)$ can be solved by a system of ordinary differential equations listed in Annex.

IV. EMPIRICAL RESULTS

We collect from Bloomberg weekly data of zero-coupon UST yields of constant maturities of 3-month, 6-month, and 1, 2, 3, 4, 5, 7, 10, 15, 20 and 30-year from January 1990 to June 2017. In estimating the model, we introduce measurement errors between the observed yields and model-implied yields. We choose the 3-month, 10-year and 30-year maturities as the benchmark maturities (i.e., assuming no measurement errors) and use these bond yields to invert the state variables. Table 1 reports the pricing errors for the non-benchmark maturities for the model. The results demonstrate that the model adequately fits for yields with all maturities, with the absolute pricing errors ranging from 4 to 46 bps.

[6] Technically, the expectation is taken under the risk-neutral probability measure Q.

Table 1. Mean and standard deviation of absolute pricing errors (in %)

	6-month	1-yr	2-yr	3-yr	4-yr	5-yr	6-yr	7-yr	8-yr	9-yr	15-yr	20-yr
Mean												
	0.11	0.18	0.28	0.27	0.25	0.23	0.18	0.14	0.09	0.04	0.36	0.46
Standard Deviation												
	0.08	0.13	0.19	0.21	0.20	0.17	0.14	0.10	0.07	0.04	0.07	0.11

Note: Absolute pricing errors are defined as absolute differences between the actual yields and the model-implied yields.

Figure 4. Estimated short rate, long-term mean and demand factor for the US Treasury yield curve

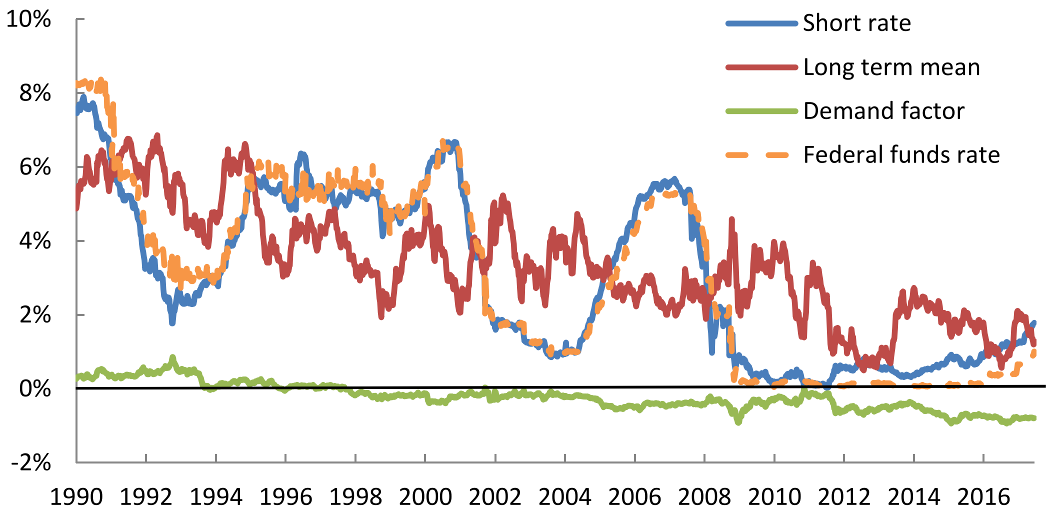

Figure 4 graphs the path of the three state variables and the FFR. It shows that the short rate tracked very closely to the FFR before the 2008 GFC. When the FFR was close to zero after 2008, the short rate also fell to close to zero but with some deviations during 2009 – mid-2011. The gap between the short rate and the FFR increased after mid-2011 at around 0.5%. The gap remained even after the Fed raised the target range of the FFR in December 2015. The demand factor L_t was quite close to zero during most of the time in the 1990s when the inflation risk was high. It dropped in the late 1990s and remained quite steady at the level of -40 bps until the 2008 GFC. In the model, a negative L_t can be interpreted as stronger-than-usual demand for the UST as safe assets, which could be partly due to the global savings gluts phenomenon since 2000. L_t edged downwards after the GFC, reflecting enhanced demand for UST due to the quantitative easing policy and the purchases by emerging market economies for their increased foreign reserves due to net capital inflows. The fall in L_t kept the effective rate $(r_t + L_t)$ close to the FFR. Given the almost zero FFR since 2008, the demand factor L_t was like a mirror image of the short rate r as shown in Figure 4.

To better understand the linkage between the demand factor and the factors related to the demand for UST, we use a simple regression of monthly frequency to identify their relationships for the Greenspan conundrum (March 2004 – March 2007) and the post GFC period (September 2008 – June 2017). As inspired by previous studies such as Longstaff (2004), we include the following explanatory variables in the regressions:

(i) Foreign holdings: This variable is the change in the amount of UST held by foreign investors and the data are obtained from the Federal Reserve Board. Increase in foreign holdings suppresses yields and hence the expected sign of foreign holdings should be negative.

(ii) VIX: We use the CBOE volatility index to gauge the global risk appetite in the financial market. An increase in the VIX index is usually associated with heightened volatility in financial markets and enhances the demand for UST as safe assets. We use the log change of the VIX index in the regression. We expect the sign of VIX to be negative.

(iii) BBB bond: This variable measures the change in the Bloomberg 5-year Industrial BBB Bond Yield Index. It is a proxy for the across-the-board search for yield behaviour in the fixed income market. A decline in the BBB bond yield index may suggest high demand for UST. As more demand for UST means the demand factor is more negative in the term structure model, the expected sign for this variable is positive.

(iv) A bond: This variable measures the change in the Bloomberg 5-year Industrial A Bond Yield Index. Investors could buy bonds with good credit ratings rather than UST as safe assets. Given that the decline in A bond yield index may be interpreted as less demand for UST, this variable should have a negative relationship with the demand factor.

(v) QE1-3: They are the zero-one dummy variables for the months of executing the three rounds of quantitative easing programs. The expected signs for these dummy variables are negative.

(vi) OT1-2: They are the zero-one dummy variables for the months of executing the two operation twists. The expected signs for these dummy variables are negative.

(vii) ΔL_{t-1}: To control for the persistence of the demand factor, a lagged value is added as an additional explanatory variable in the regression.

Table 2 reports the regression results. The first and second columns of the table show the results for the Greenspan Conundrum and the post GFC period respectively. With the exception of the QE3 dummy, all the explanatory variables have the expected signs and are statistically significant. The reason that the dummy variable for QE3 is insignificant is probably attributable to the fact that QE3 was widely anticipated by financial markets ahead of the actual announcement in 2012.

Table 2. Results from regression of ΔL

	Greenspan Conundrum (Mar 2004 – Mar 2007)		Post GFC (Sep 2008 – Jun 2017)	
Variables	Coefficients	*t*-stat	Coefficients	*t*-stat
Constant	-2.99E-05	-1.142	8.76E-05*	1.838
ΔL_{t-1}	0.3736***	7.874	0.2680***	3.825
ΔBBB_t	0.0015***	4.775	0.0021**	2.689
ΔA_t	-0.0011***	-4.118	-0.0228**	-2.529
ΔForeign $Holdings_{t-1}$	-5.33E-07**	-2.851	-1.99E-06***	-2.921
ΔVIX_{t-1}	-0.0003*	-1.944	-0.0004*	-1.824
$QE1_t$			-0.0021***	-8.165
$QE2_t$			-0.0012***	-6.215
$QE3_t$			8.83E-05	0.629
$OT1_t$			-0.0012***	-7.988
$OT2_t$			-0.0004***	-9.509
Adj. R^2	0.273		0.367	
No. of Observations	37		106	

Note: The table presents the results of estimating ΔL on a monthly basis. ***, **, and * respectively indicate significance at the 1%, 5%, and 10% level. Newly-West t-statistics are reported. Adjusted R^2 estimates are provided in the row labelled "Adj. R^2". ΔBBB and ΔA are the monthly changes in the Bloomberg 5 year US industrial BBB and A corporate bond yield indexes respectively in percentage points. Δ(Foreign Holdings) is the monthly change in the total amount of foreign holdings of US Treasury bonds measured in billions of dollars. ΔVIX is the log-difference of monthly average of VIX. QE1, QE2 and QE3 are the dummy variables for the months of executing three quantitative easing programs. OT1 and OT2 are the dummy variables for the months of executing two rounds of operation twist.

V. CONCLUSION

This study investigates the driving forces behind the compressed UST yields and this picture may evolve as the Fed normalises monetary policy in coming years. The forces can be taken as a demand factor that resembles the convenience yield that investors are willing to forego in exchange for owning a highly liquid and safe asset. We then use a term structure model to extract this demand factor from the UST yield curve. Unlike previous studies that typically give a point estimate of convenience yield over a period of time, our estimate of the demand factor is available at weekly frequency, and this enables us to run regression model to identify its driving forces. In summary, our estimation results suggest that the demand factor was underscored by a number of market forces during the Greenspan Conundrum and the post-GFC period. Specifically, foreign purchases, safe asset demand and search for yield were key forces keeping UST yields low during both episodes. QE1, QE2 and OT (but not QE3) also contributed to suppressing yields in the post GFC period. As the Fed has recently announced its plan to normalise its balance sheet, UST yields will be affected in the context of the future path of the Fed's balance sheet.

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Annex. The bond pricing formula and the estimation

The three equations of the term structure model are

$$r_t = \kappa(\theta_t - r_t) \quad + \quad \overline{r_t}\ \ Z_r \tag{A1}$$
$$\theta_t = \alpha(\beta - \theta_t) \quad + \quad \overline{\theta_t}\ \ Z \tag{A2}$$
$$L_t = -\ \ L_t \quad + \gamma\ \ Z_L \tag{A3}$$

The price of a zero-coupon bond with a maturity at time $\tau = T -$ is given by:

$$P_t(\tau, r, \theta, L) = E_t\ \ exp\ \ -\ \ _t\ (r_t + L_t) \tag{A4}$$

To preserve analytical tractability, we set the market price of risk as $(\lambda_r\sqrt{r}, \lambda\ \ \sqrt{\theta}, \lambda_L)$ for the state variables (r, θ, L) respectively. With the assumed functional form for risk premium, we can rewrite Eqs. (A1)-(A3) under the risk-neutral measure Q, and the conditional expectation in Eq. (A4) can be calculated by solving the following partial differential equation:

$$(r + L)P = \frac{\partial P}{\partial} + \frac{1}{2}\ \ {}^2 r\frac{\partial^2 P}{\partial r^2} + \left(\kappa\theta - (\kappa + \lambda_r\ \ )\right)\frac{\partial P}{\partial r} + \frac{1}{2}\ \ {}^2\theta\frac{\partial^2 P}{\partial \theta^2}$$
$$+ \left(\alpha\beta - (\alpha + \lambda\ \ \ )\right)\frac{\partial P}{\partial \theta} + \frac{1}{2}\gamma^2\frac{\partial^2 P}{\partial L^2} - (\ \ + \lambda_L\gamma)\frac{\partial P}{\partial L}$$

It can be solved that the solution of the above partial differential equation is

$$P_t(\tau, r, \theta, L) = exp[A(\tau) - B(\tau)r_t - C(\tau)\theta_t - D(\tau)L_t]$$

where the coefficient functions $A(\tau), B(\tau), C(\tau), D(\tau)$ can be solved by a system of ordinary differential equation as follows:

$$\frac{A(\tau)}{\tau} = -\alpha\beta\ C(\tau) + \frac{1}{2}\gamma^2 D^2(\tau) + \lambda_L\gamma D(\tau)$$
$$\frac{B(\tau)}{\tau} = 1 - \frac{1}{2}\ \ {}^2 B^2(\tau) -\ (\kappa + \lambda_r\ \ )B(\tau)$$
$$\frac{C(\tau)}{\tau} = \kappa B(\tau) - \frac{1}{2}\ \ {}^2 C^2(\tau) -\ (\alpha + \lambda\ \ \ )C(\tau)$$
$$\frac{D(\tau)}{\tau} = 1 -\ \ D(\tau)$$

for $\tau \geq 0$ and $A(0) = B(0) = C(0) = D(0) = 0$.

We estimate the model using the closed-form maximum likelihood method developed by Ait-Sahalia and Kimmel (2010). Table A1 reports the parameter estimates for the model.

Table A1. Maximum likelihood estimates

	Estimates	t-ratios
Short rate process (r)		
mean reversion (κ)	0.3924	22.16
volatility ( )	0.0622	4.81
risk premium (λ_r)	-2.4036	-3.89
Long-term mean process (θ)		
mean reversion (α)	0.1076	2.61
long-term mean (β)	0.0603	4.51
volatility ( )	0.0537	2.84
risk premium (λ_θ)	0.1746	0.18
Exogenous process (L)		
drift ( )	-0.0634	-5.58
volatility (γ)	0.0021	1.91
risk premium (λ_L)	-0.0169	-0.04

Note: The sample is weekly from January 1990 to June 2017.