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HKIMR Working Paper No.16/2003

August 2003

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Intervention in the Foreign Exchange Market in a Model with Noise Traders

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Abstract

In this paper we analyze the effectiveness of sterilized interventions in the foreign exchange market. We use a model in which chartists and fundamentalists interact. This model produces speculative noise which leads to systematic deviations of the exchange rate from its fundamentals. In such an environment, interventions can be effective in reducing the noise. It does this by reducing the profitability of noise trading.

* This paper was prepared while the authors were visiting the Hong Kong Institute for Monetary Research during September 2002. The authors are grateful for the financial support and for the excellent research environment provided by the Institute.

The views expressed in this paper are those of the authors and do not necessary reflect those of the Hong Kong Institute for Monetary Research, its Council of Advisors, or Board of Directors.

1. Introduction

The "Washington consensus" of the 1990s about the desirable features of the international economic order included the view that countries allowing open goods and capital markets would have no other choice than to move to a pure floating exchange rate regime or else to enter into a monetary union. In this view intermediate regimes characterized by management of the exchange rate should be avoided because they are inherently unstable and sooner or later lead to a crisis. This consensus was reached mainly as a result of the collapses of fixed exchange rate regimes during the 1990s, first in Europe when the EMS disintegrated during 1992-93 and later in South-East Asia during 1997-98. This view was also given intellectual respectability by the development of theoretical models of exchange crises. In these models, rational agents trigger the collapse of the fixed exchange rate when the authorities fail to discipline monetary and fiscal policies to the rigours imposed by the markets, or worse when these rational agents create the crisis by the very fact that they predict it to happen.

Despite the power of this analysis many countries have refused to follow the prescriptions of the "Washington consensus", and have continued to manage their exchange rate, out of "fear of floating" (see Calvo and Reinhart 2000). In Asia in particular we observe that after the crises of 1997-98 many countries have gradually narrowed the size of the exchange rate fluctuations by active management of the exchange market (McKinnon and Schnabl 2002). There are several reasons for this creeping return to pegged exchange rates. First, in the case of Asia, the dollar maintains its dominating position as an invoicing currency despite the increasing intra-Asian trade. As a result, pegging to the dollar allows these countries to stabilize their exchange rates among themselves. A failure to do so carries the risk of "beggar-thy-neighbour" devaluations. Second, there is what Eichengreen and Hausmann (1999) have called the "original sin": because of underdeveloped capital markets, Asian countries lack the ability to borrow in their own currency. This also makes it difficult for stabilizing speculation to operate. As a result, there are few if any private market makers. Free float would, under those circumstances, lead to excessive volatility. Intervention in the foreign exchange market is seen as a way to stabilize the market (McKinnon and Schnabl 2002).

The academic models of the exchange rates have been based on the rational expectations paradigm, in which agents know the true distributions of the shocks in the fundamentals of the exchange rate. With this knowledge they force the exchange rate to reflect these fundamentals at each point in time. In such a world there is very little the authorities can do to influence the exchange rate without changing the current or future fundamentals. Sterilized intervention, i.e. intervention that keeps the current fundamentals unchanged is mostly futile, except if it is seen as a signal of a future change of (policy induced) fundamentals (see Dominguez and Frankel 1993). In this view only non-sterilized intervention can be effective.[1]

[1] There are models (the portfolio balance models) that allow for the possibility of sterilized intervetion to have an effect on the exchange rate through the portfolio channel. This comes from the fact that sterilized intervention changes the composition of portfolios and thus the risk premia. There is a consensus however that this effect is weak and cannot easily be expoited by the authorities (see Dominguez and Frankel 1993).

Despite this analysis, many central banks like those in Asia, that actively manage their exchange rates, overwhelmingly use sterilized interventions. Thus not only is there a strong desire to peg, but in addition, central banks use intervention policies that academic models predict to be relatively ineffective. One way to react to this official stubborness is to shrug our academic shoulders and to predict that these attempts at managing the exchange markets will prove to be futile. Another way is to analyze our own models critically. There may be something in the foreign exchange markets not captured by our models that leads the monetary authorities to use sterilized interventions and to have more success than predicted.

In this paper we present a model of the foreign exchange markets in which heterogeneous agents hold different beliefs about the future exchange rate. These agents are not rational in the sense given to this notion in mainstream models. Instead they use limited information sets to make forecasts. Rationality comes in the model in that these agents evaluate the profitability of these forecasting rules and decide ex post whether or not to switch to the more profitable one. The interaction of these heterogeneous agents leads to a rich dynamics in which the exchange rate is dissociated (disconnected) from its fundamentals most of the time, and in which periods of tranquility and turbulence alternate in unpredictable fashion. We will analyze the rationale for sterilized intervention in the context of such a model. We will show that sterilized intervention has the capacity to change the nature of the dynamics of such a market.

The remainder of the paper is organized as follows. In sections 2 to 7 we present the model of the exchange market and we describe its main features. We also argue that this model captures the essential empirical features of the foreign exchange market, and is therefore suitable to analyze the effect of intervention policies. In section 10 we model the intervention policies and in section 11 we describe the effects of these intervention policies.

2. The effectiveness of sterilized intervention: a brief review of the literature

While the consensus about the effectiveness of non-sterilized intervention has always been strong, the same cannot be said about the effectiveness of sterilized intervention.

Two major channels have been identified through which sterilized intervention can potentially affect the exchange rate, i.e. the *portfolio balance channel* (PBC) and the *signalling channel* (see Sarno and Taylor 2001 for a brief but exaustive survey). According to the PBC, sterilized intervention can affect the exchange rate because it changes the investors' portfolio composition. However, the effect of the intervention will occur only if there is some degree of imperfect substitutability of the assets held in the portfolio (see Branson 1983) and/or the Ricardian equivalence does not hold (Barro 1974). In a world of increasing capital mobility and perfect substitutability among domestic and foreign assets the importance of this channel will tend to be reduced. In the limit when foreign and domestic assets are perfect substitutes, sterilized intervention will not affect the exchange rate anymore.

Sterilized intervention can still be effective under perfect substitutability through the signalling channel (Mussa 1981, Dominguez and Frankel 1993). The idea is that economic agents consider the exchange rate intervention as a signal about the future monetary policy. In other words, they interpret this intervention to lead to future changes in fundamentals. It is important to stress here that the implicit presumption is that the central bank backs up intervention with a subsequent change in the monetary policy as expected by the agents. If this is not the case, the central bank loses its credibility and future intervention will be unsuccessful. This leads us to consider a possible drawback of the signalling channel. If the central bank has a weak reputation or a low credibility, then the signalling might fail or even give arise to a “perverse” reaction from the market. But such a drawback can be mitigated by the practice of coordinated intervention, i.e. two or more central banks intervene in the market simultaneously in support of the same currency. In the case of coordinated intervention, the effectiveness of intervention increases since the agents are willing to follow a multiple consistent signal rather than a single signal (see Dominguez and Frankel 1993a). The *coordination channel* can be seen as representing a third channel through which the central bank can affect the exchange rate.

A closely related strand of literature addresses the issue of the existence of a link between profitability of technical trading rules and central bank intervention in the foreign exchange market.[2] This literature investigates whether simple trading rules have some predictive power and whether the forecastibility of the exchange rate is influenced by central bank interventions. The general results are that the exchange rate predictability is dramatically reduced when intervention periods are excluded from the analysis (see Le Baron 1996, 1999). However, such a link between predictability of exchange rates and profitability of technical trading rules has been found to be weak or not existent by other researchers (see Neely and Weller 2001). According to the latter, the predictability of the exchange rate is mainly due to the existence of trends in the exchange rate that intervention tends to reverse. It is important to note that these studies do not consider intervention acting as a signal of future monetary policies. Thus they do not take into account that if the signalling channel works, intervention policies can affect the exchange rate because these policies carry information that current and past exchange rates do not have.

3. The model

In this section we develop a simple non-linear model of the exchange rate. We assume agents of different types i depending on their beliefs about the future exchange rate. Each agent can invest in two assets, a domestic and a foreign one. The agents’ utility function can be represented by the following equation:

$$U(W^i_{t+1}) = E_t(W^i_{t+1}) - \frac{1}{2}\mu V^i(W^i_{t+1}) \tag{1}$$

2 There is now a considerable amount of evidence to suggest that technical trading rules are profitable since they can earn significant excess returns in the foreign exchange market (see Taylor and Allen 1992 and Neely and Weller 1999).

where W_{t+1}^{i} is the wealth of agent of type i at time $t+1$, E_t is the expectation operator, μ is the coefficient of risk aversion and $V^i(W_{t+1}^i)$ represents the conditional variance of wealth of agent i. The wealth is specified as follows:

$$W_{t+1}^{i} = (1+r^{*})\, s_{t+1} d_t^i + 1 + r\left(W_t^i - s_t d_t^i\right) \tag{2}$$

where r and r^* are respectively the domestic and the foreign interest rates, $s_t + 1$ is the exchangerate at time $t+1$, $d_{i,t}$ represents the holdings of the foreign assets by agent of type i at time t. Thus, the first term on the right-hand side of 2 represents the value of the foreign portfolio in domestic currency at time $t+1$ while the second term represents the value of the domestic portfolio at time $t+1$.

Substituiting equation 2 in 1 and maximizing the utility with respect to $d_{i,t}$ allows us to derive the optimal holding of foreign assets by agents of type i:

$$d_{i,t} = \frac{(1+r^{*})\, E_t^i\left(s_{t+1}\right) - (1+r)\, s_t}{\mu \sigma_{i,t}^2}. \tag{3}$$

The market demand for foreign assets at time t is the sum of the individual demands, i.e.:

$$\sum_{i=1}^{N} n_{i,t} d_{i,t} = D_t \tag{4}$$

where $n_{i,t}$ is the number of agents of type i.

Market equilibrium implies that the market demand is equal to the market supply X_t. The latter increases (declines) when the current account is positive (negative) and when the monetary authorities sell (buy) foreign assets. We assume here that X_t is exogenous. (When we introduce interventions this variable will become endogenous). Thus,

$$X_t = D_t. \tag{5}$$

Substituting the optimal holdings into the market demand and then into the market equilibrium equation and solving for the exchange rate s_t yields the equilibrium exchange rate:

$$s_t = \left(\frac{1+r^{*}}{1+r}\right) \frac{1}{\sum\limits_{i=1}^{N} \frac{n_{i,t}}{\sigma_{i,t}^2}} \left[\sum_{i=1}^{N} n_{i,t} \frac{E_t^i\left(s_{t+1}\right)}{\sigma_{i,t}^2} - \mu \frac{X_t}{1+r}\right] \tag{6}$$

In order to model the expectations formation we assume that there are two types of agents: chartists and fundamentalists. As a result equation 6 specializes to:

$$s_t = \left(\frac{1+r^{*}}{1+r}\right) \frac{1}{\left(\frac{n_{f,t}}{\sigma_{f,t}^2} + \frac{n_{c,t}}{\sigma_{c,t}^2}\right)} \left[n_{f,t} \frac{E_t^f\left(s_{t+1}\right)}{\sigma_{f,t}^2} + n_{c,t} \frac{E_t^c\left(s_{t+1}\right)}{\sigma_{c,t}^2} - \mu \frac{X_t}{1+r}\right] \tag{7}$$

Thus the exchange rate is determined by the expectations of fundamentalists and chartists about the future exchange rate. These forecasts are weighted by their respective variances.

We now specify how fundamentalists and chartists form their expectations of the future exchange rate. Then we will specify how they take into account the risk as measured by the variances.

The fundamentalists base their forecast on a comparison between the market and the fundamental exchange rate, i.e. they forecast the market rate to return to the fundamental rate in the future. In this sense they use a negative feedback rule that introduces a mean reverting dynamics in the exchange rate. The speed with which the market exchange rate returns to the fundamental is assumed to be determined by the speed of adjustment in the goods market. Thus, the forecasting rule for the fundamentalists is:

$$E_t^f\left(\Delta s_{t+1}\right) = -\psi_t\left(s_t - s_t^*\right) \tag{8}$$

where s_t^* is the fundamental exchange rate at time t, which is assumed to follow a random walk.

The parameter ψ_t is determined by the speed of adjustment in the goods market. There is a large body of empirical evidence indicating that the speed of adjustment in the goods market follows a non-linear dynamics, i.e. the speed with which prices adjust towards equilibrium depends positively on the size of the deviation of the exchange rate from its fundamental value (misalignment) (Kilian and Taylor 2001, Taylor, Peel and Sarno 2001, Nobay and Peel 1997). We will assume that the fundamentalists take into account this non-linear adjustment process in making their forecasts. We specify ψ_t as follows:

$$\psi_t = \theta \mid s_t - s_t^* \mid \tag{9}$$

The economics behind this non-linear specification is that in order to profit from arbitrage opportunities in the goods market, some fixed investment must be made, e.g. trucks must be bought, planes be chartered, etc. These investments become profitable with sufficiently large deviations from the fundamental exchange rate. Note that we do not model the goods market explicitly but we assume that in order to form their expectations about the exchange rate, the fundamentalists take into account the dynamics of the goods market and the speed of adjustment of goods prices.

The chartists forecast the future exchange rate by extrapolating past exchange rate movements. Their forecasting rule can be specified as:

$$E_t^c\left(\Delta s_{t+1}\right) = \beta \sum_{i=0}^{T} \alpha_i \Delta s_{t-i} \tag{10}$$

Thus, the chartists compute a moving average of the past exchange rate changes and they extrapolate this into the future exchange rate change. The degree of extrapolation is given by the parameter β. Note that in contrast to the fundamentalists they do not take into account information concerning the fundamental exchange rate. In this sense they can be considered to be pure noise traders.

Our choice to introduce chartists' rules of forecasting is based on empirical evidence. The evidence that Chartism is used widely to make forecasts is overwhelming (see Cheung and Chinn 1989, Taylor and Allen 1992). Therefore, we give a prominent role to chartists in our model. It remains important, however, to check if the model is internally consistent. In particular, the chartists' forecasting rule must be shown to be profitable within the confines of the model. If these rules turn out to be unprofitable, they will not continue to be used. We return to this issue when we let the number of chartists be determined by the profitability of the chartists' forecasting rule.

We now analyze how fundamentalists and chartists evaluate the risk. The latter is measured by the variance terms in equation 7, which we define as the weighted average of the squared (one period ahead) forecasting errors made by chartists and fundamentalists, respectively. Thus,

$$\sigma_{i,t} = \sum_{k=1}^{\infty} \gamma_k \left[E_{t-k}^{i} \left(s_{t-k+1} \right) - s_{t-k+1} \right]^2 \qquad (11)$$

where γ_k are geometrically declining weights.

However fundamentalists and chartists perceive the risk in a different way. In particular the fundamentalists are assumed to take into account the deviation of the exchange rate from the fundamental in addition to the forecasting error. We will call the deviation of the market exchange rate from fundamentals, the misalignment. Thus the fundamentalists' risk term can be written as:

$$\sigma_{f,t} = \frac{\sum_{k=1}^{T} \gamma_k \left[E_{t-k}^{f} \left(s_{t-k+1} \right) - s_{t-k+1} \right]^2}{\left(s_{t-1} - s_{t-1}^{*} \right)^2} \qquad (12)$$

where $\left(s_{t-1} - s_{t-1}^{*} \right)$ is the misalignment.

The logic behind this specification is that the fundamentalists consider the fundamental exchange rate as a benchmark. The larger is the misalignment the less the fundamentalists will attach importance to the short-term volatility as measured by the one-period ahead forecasting error. Put differently, the larger is the deviation of the exchange rate from its fundamental, the more confident the fundamentalists are about the probability of a return towards the fundamental rate. As a result, they perceive the risk of using a fundamentalist forecasting rule to decline when the misalignment increases. In contrast the chartists do not take into account the misalignment.

We now specify the dynamics that governs the number of chartists and fundamentalists, namely $n_{c,t}$ and $n_{f,t}$. In order to do so, we describe how the number of chartists and fundamentalists changes from period t-1 to period t:

$$n_t^c = n_{t-1}^c + n_{t-1}^f p_t^{fc} - n_{t-1}^c p_t^{cf} \tag{13}$$

$$n_{f,t} = n_{f,t-1} + n_{c,t-1} p_t^{cf} - n_{f,t-1} p_t^{fc} \tag{14}$$

where $n_{c,t}$ and $n_{f,t}$ are the number of chartists and fundamentalists in period t; p_t^{cf} represents the fraction of the chartists who decide to become fundamentalists in period t, and p_t^{fc} is the fraction of the fundamentalists who decide to become chartists in period t.

These fractions are assumed to be a function of the relative utilities of chartism and fundamentalism. These utilities depend on the profitability of the forecasting rules and the risk associated with their use. The fractions are specified as follows:[3]

$$p_t^{fc} = \varsigma \frac{exp\left(U_{c,t-1}\right)}{exp\left(U_{c,t-1}\right) + exp\left(U_{f,t-1}\right)} \tag{15}$$

$$p_t^{cf} = \varsigma \frac{exp\left(U_{f,t-1}\right)}{exp\left(U_{c,t-1}\right) + exp\left(U_{f,t-1}\right)} \tag{16}$$

where $U_{c,t}$ -1 and $U_{f,t}$ -1 are the utility functions of the chartists and fundamentalists in period t-1. The parameter ς measures the sensitivity of the decision to switch with respect to the relative profits adjusted for the risk. Thus this parameter can be interpreted as a measure of the degree of inertia in the decision of switching. A low ς implies a high degree of inertia. Note also that $0 < \varsigma < 1$. Equations 15 and 16 can be interpreted as follows. When the utility associated with using chartist information increases relative to the utility of using fundamentalist information the agents tend to switch from fundamentalism to chartism and vice versa. Taking into account the specification of the utility function (equation 1), we can rewrite equations 15 and 16 as follows:

$$p_t^{fc} = \varsigma \frac{exp\left(\pi_{c,t-1} - \mu\sigma_{c,t-1}^2\right)}{exp\left(\pi_{c,t-1} - \mu\sigma_{c,t-1}^2\right) + exp\left(\pi_{f,t-1} - \mu\sigma_{f,t-1}^2\right)} \tag{17}$$

$$p_t^{cf} = \varsigma \frac{exp\left(\pi_{f,t-1} - \mu\sigma_{f,t-1}^2\right)}{exp\left(\pi_{c,t-1} - \mu\sigma_{c,t-1}^2\right) + exp\left(\pi_{f,t-1} - \mu\sigma_{f,t-1}^2\right)} \tag{18}$$

Thus, the decision to switch depends on the relative profits adjusted for the risk. When the risk adjusted profits of chartist rules increase relative to the risk adjusted profits of fundamentalist rules, a certain fraction of fundamentalists decide to switch towards the use of chartist rules. Agents make a profit when they correctly forecast the direction of the exchange rate movements. They make a loss if they wrongly predict the direction of its movements. The profit (the loss) they make equals the one-period return of investing $1.

3 Such a specification is often found in discrete choice models. See e.g. Brock and Hommes (1996).

4. The model with transactions costs

There is an increasing body of theoretical literature stressing the importance of transactions costs as a source of non-linearity in the determination of the exchange rate (Dumas 1992, Sercu, Uppal and Van Hulle 1995, Obstfeld and Rogoff 2000). The importance of transaction costs has also been confirmed empirically (Taylor, Peel and Sarno 2001, Kilian and Taylor 2001). Therefore, we will develop a version of the previous model in which the transaction costs play a role.

We take the view that if transaction costs in the goods markets exist, the fundamentalists take this information into account. Therefore, if the exchange rate is within the transaction costs band the fundamentalists behave differently than if the exchange rate moves outside the transaction costs band.

Consider the first case, when the exchange rate deviations from the fundamental value are smaller than the transaction costs. In this case the fundamentalists know that arbitrage in the goods market does not apply. As a result, they expect the changes in the exchange rate to follow a white noise process. The best they can do is to forecast no change. More formally,

$$\text{when } \mid s_t - s_t^* \mid < C, \qquad \text{then } E_t^f(\Delta s_{t+1}) = 0.$$

In the second case, when the exchange rate deviation from its fundamental value is larger than the transaction costs C (assumed to be of the 'iceberg' type), then the fundamentalists follow the same forecasting rule as in equation 8. More formally,

$$\text{when } \mid s_t - s_t^* \mid > C \text{ holds, then equation 8 applies.}$$

This formulation implies that when the exchange rate moves outside the transaction costs band, market inefficiencies other than transaction costs continue to play a role. As a result, these inefficiencies prevent the exchange rate from adjusting instantaneously. In our model these inefficiencies are captured by the fact that the speed of adjustment in the goods market is not infinite (equation 9).

5. Solution of the model

In this section we investigate the properties of the solution of the model. We first study its deterministic solution. This will allow us to analyze the characteristics of the solution that are not clouded by exogenous noise. We use simulation techniques since the non-linearities do not allow for a simple analytical solution. We select "reasonable" values of the parameters, i.e. those that come close to empirically observed values.[4]

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4 In De Grauwe and Grimaldi (2002) we performed an extensive sensitivity analysis to check the robustness of the results.

We first concentrate on the fixed point solutions of the model. We find that for a relatively wide range of parameters the solution converges to a fixed point (a fixed-point attractor). However, there are many such fixed points (attractors) to which the solution converges depending on the initial conditions. We illustrate this feature in figure 1 where we plot the fixed point solutions (attractors) as a function of the different initial conditions. On the horizontal axis we set out the different initial conditions. These are initial shocks to the deterministic system. The vertical axis shows the solutions corresponding to these different initial conditions. Note the complex pattern of these fixed point solutions, with many discontinuities. This has the implication that a small change in the initial condition can have a large effect on the solution. This feature lies at the heart of some of the results that are obtained with this model relating to the unpredictability of the effect of shocks in exogenous variables. We return to this phenomenon in section 6.

It is also interesting to point out the existence of critical points. These are often obtained close to the limits of the transactions cost band. A continuous line is then broken into a cluster of points. These have a fractal nature, in that they have infinite self-replicating characterisitics.

6. Sensitivity analysis

In this section we perform a sensitivity analysis. We do this by showing diagrams that relate the solutions to different values of important parameters of the model. We concentrate on the extrapolation parameter used by the chartists, as this turns out to be a crucial parameter determining the nature of the solutions.[5] Figures 2 shows an example of such a diagram. It is analogous to the so-called bifurcation diagram. On the vertical axis we set out different values of the extrapolation parameter. On the vertical axis we show the solutions for the exchange rate. This is the exchange rate obtained after 1000 periods, given an initial shock. We observe the following. For values of $\beta \leq 0.8$ we obtain a continuous line. This means that there is a unique fixed point solution for each value of β. When β is approximately 0.8 we have a critical point where the continuous line breaks into a fractal pattern. We still have fixed-point solutions, though. The continuous line, however, is fractured in such a way that for each value of β only one solution is found. When β reaches a value of approximately 0.9, we enter the chaotic region. This is characterized by infinitely many solutions for each value of β. These points correspond to strange attractors within which the exchange rate then travels. Note that we do not obtain bifurcations in this model, like the Hoppf bifurcation. The transition to chaos is abrupt. We do find that increasing the value of β can lead us in and out of the chaotic region.[6]

5 In the appendix we do a similar analysis varying other parameters.

6 Note that different initial conditions lead to different routes to the chaotic region.

It is also important to analyze the dynamics of the weights of chartists and fundamentalists. We find that when the exchange rate converges towards a fixed-point attractor the weights of chartists and fundamentalists converge to 0.5. This can be explained as follows. When the exchange rates reach a fixed-point solution, chartists and fundamentalists expect no further change.[7] Therefore, they do not buy or sell, and thus make neither profits nor losses. As a result, the profit related selection rule of the model (see equations 11 and 12) assures that they will be equally represented in the market.

Things are very different when the exchange rate follows a chaotic pattern. In this case the chartists' and fundamentalists' weights will show cyclical movements. In general the chartists' weight will tend to fluctuate around a value larger than 0.5 provided the chartist extrapolation is not too high. We come back to this feature in section 8 where we analyze the dynamics of the weights of chartists and fundamentalists in more detail.

The empirical evidence in favour of deterministic chaos is not very strong. Sometimes deterministic chaos has been detected in the data, but most often no such dynamics has been found (Granger 1994, Guillaume 1996, Schittenkopf, Dorffner and Dockner 2001). This suggests that we should restrict the analysis of the model to parameter configurations that do not produce deterministic chaos. Typically this implies restricting the extrapolation parameter used by chartists not to exceed 0.9 - 1. The borderline between fixed-point solutions and deterministic chaos remains interesting though. As we have seen, even with an extrapolation parameter between 0.9 and 1 different initial conditions lead to switches in and out of chaos, making it difficult to detect deterministic chaos from the data.[8]

In the next section we restrict the analysis of the model for parameter values that do not lead to deterministic chaos. We will show that in combination with stochastic shocks this model is capable of producing a complex dynamics that exhibits many of the features of chaotic dynamics despite the fact that the deterministic solutions of the model are fixed points.[9]

7. The stochastic version of the model

We now introduce stochastic disturbances to the model. In our model these disturbances affect the fundamental, which is assumed to be a random walk. In addition, as can be seen from equation (8), there is exogenous noise leading to forecast errors of chartists and fundamentalists. We simulate the model with a certain combination of parameter values that we refer to as the "standard case". This includes setting c=5, β =0.5 and θ=0.2. (Similar results are obtained for a wide range of parameter values.)

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7 Things are a little more complicated. When transactions costs are zero, we obain non-zero fixed points. Since the fundamental exchange rate was normalized at zero, this means that the market exchange rate is permanently different from the fundamental rate. Thus in the steady state the fundamentalists expect a constant appreciation (depreciation). However, because of the risk involved, fundamentalists do not act on this. The difference between the market rate and the fundamental rate can be considered to be the risk premium. Our model generates risk premia whose value depends on the initial conditions.

8 Note that this switching in and out of chaos depending on the initial conditions tends to disappear for sufficiently high values of β.

9 This result is similar to the one obtained by Malliaris and Stein (1999).

A first feature of the solution of the stochastic version of the model is the sensitivity to initial conditions. In order to show this, we first simulated the model with the "standard" parameter values and then we simulated the model with the same parameters setting but with a slightly different initial condition. In both cases we used identical stochastic disturbances. We show the time paths of the (market) exchange rate in figure 5.

We observe that after a certain number of periods the two exchange rates start following a different path. This result is related to the presence of many fixed-point attractors in the deterministic part of the model, which are themselves dependent on the initial conditions (see figure 1, which shows how slight differences in initial conditions can lead to fixed-point attractors that are very far apart). As a result, the two exchange rates can substantially diverge because they are attracted by fixed-points that are far away from each other. The nice aspect of this is that we obtain a result that is typical for chaotic systems, however, without chaos being present in the deterministic part of the model. The combination of exogenous noise and a multiplicity of fixed-point attractors creates chaos-like dynamics. As will be seen later, interventions in the foreign exchange markets have the capacity to alter this feature of the model.

A second feature of the model relates to the way shocks in the fundamental exchange rate are transmitted into the market exchange rate. In linear models a permanent shock in the fundamental has a predictable effect on the exchange rate, i.e. the coefficient that measures the effect of the shock in the fundamental on the exchange rate converges after some time to a fixed number. Things are very different in our non-linear model. We illustrate this by showing how a permanent increase in the fundamental is transmitted to the exchange rate. We assumed that the fundamental rate increases by 10, and we computed the effect on the exchange rate by taking the difference between the exchange rate with the shock and the exchange rate without the shock. In a linear model we would find that in the long run the exchange rate increases by 10. This is not the case in our model. We present the evidence in figure 6 where we show the effect of the same permanent shock of 10 in the fundamental rate on the exchange rate. The simulations are done assuming exactly the same stochastics in the scenario with and without the permanent shock in the fundamental exchange rate. Thus, there is no exogenous noise in the model that could blur the transmission process from the fundamental rate to the exchange rate.

The most striking feature of these results is that the effect of the permanent shock does not converge to a fixed number. In fact, it follows a complex pattern. Thus, in a non-linear world it is very difficult to predict what the effect will be of a given shock in the fundamental, even in the long run. Such predictions can only be made in a statistical sense, i.e. our model tells us that on average the effect of a shock of 10 in the fundamental will be to increase the exchange rate by 10. In any given period, however, the effect could deviate substantially from this average prediction.[10]

[10] These results are maintaned for different parameter configurations.

The importance of the initial conditions for the effect of a permanent shock in the fundamental can also be seen by the following experiment. We simulated the same permanent shock in the fundamental but applied it in two different time periods. In the first simulation we applied the shock in the first period; in the second simulation we applied it in the next period. The exogenous noise was identical in both simulations. Thus the only difference is in the timing of the shock. We show the results in figure 6.

We observe that the small difference in timing changes the whole future history of the exchange rate. As a result, the effect of the shock measured at a particular point in time can be very different in both simulations. Thus history matters. The time at which the permanent shock occurs influences the effects of the shock.

Our results help to explain why in the real world it appears so difficult to predict the effects of changes in the fundamental exchange rate on the market rate, and why these effects seem to be very different when applied in different periods. In fact this is probably one of the most intriguing empirical problems. Economists usually explain the difficulty of forecasting the effects of a particular change in one exogenous variable (e.g. an expansion of the money stock) by invoking the *ceteris paribus* hypothesis., i.e. there are usually other exogenous variables changing unexpectedly, preventing us from isolating the effect of the first exogenous variable. In our model the uncertainty surrounding the effect of a disturbance in an exogenous variable is not due to the failure of the *ceteris paribus* hypothesis. No other exogenous variable is allowed to change. The fact is that the change in the exogenous variable occurs at a particular time, which is different from all other times. Initial conditions (history) matters in forecasting the effect of shocks. Since each initial condition is unique, it becomes impossible to forecast the effect of a shock at any given point in time with any precision.

Finally, it should be stressed that the uncertainty about the effect of a permanent shock in the fundamental only holds in a particular environment that is related to a low variance of the noise. In a later section we will analyze how different environments concerning the variance of shocks affect the results.

8. Empirical relevance of the model

In this section we analyze how well our model mimics the empirical anomalies and puzzles that have been uncovered by the flourishing empirical literature. The way we proceed is to calibrate the model in such a way that it best replicates the observed statistical properties of the exchange rate movements. One result of this calibration effort is that we reject the parameter sets that produce deterministic chaos, because when the latter occurs we fail to reproduce the observed statistical properties of exchange rate movements. Put differently, we find that chaotic dynamics does not provide for a good explanation of the empirical puzzles detected in the foreign exchange markets. This calibration effort can be considered as a way to “discipline” the model, whereby the discipline is provided by the observed statistical properties of the exchange rates that the model should replicate.

8.1. The disconnect puzzle

The first and foremost empirical puzzle has been called the "disconnect" puzzle (see Obstfeld and Rogoff 2000), i.e. the exchange rate appears to be disconnected from its underlying fundamentals most of the time. It was first analyzed by John Williamson (1985) who called it the 'misalignment problem'. This puzzle was also implicit in the celebrated Meese and Rogoff studies of the early 1980s documenting that there is no stable relationship between exchange rate movements and the news in the fundamental variables. Goodhart (1989) and Goodhart and Figlioli (1991) found that most of the changes in the exchange rates occur when there is no observable news in the fundamental economic variables. This finding contradicted the theoretical models (based on the efficient market hypothesis), which imply that the exchange rate can only move when there is news in the fundamentals.

Our model is capable of mimicking this empirical regularity. In figure 7 we show the market exchange rate and the fundamental rate for a combination of parameters that does not produce deterministic chaos. (Our results hold equally well for a large set of parameter values including those that produce deterministic chaos).

We observe that the market rate can deviate from the fundamental value substantially and in a persistent way. Moreover, it appears that the exchange rate movements are often disconnected from the movements of the underlying fundamental. In fact, they often move in opposite directions.

We show the nature of the disconnect phenomenon in a more precise way by applying a cointegration analysis to the simulated exchange rate and its fundamental using the same parameter values as in figure 7 for a sample of 8000 periods. We found that there is a cointegration relationship between the exchange rate and its fundamental. Note that in our setting there is only one fundamental variable. This implies that no bias from omitted variables can occur.

In the next step we specify an EC model in the following way:

$$\Delta s_t = \mu \left(s_{t-1} - \gamma s^*_{t-1} \right) + \sum_{i=1}^{n} \lambda_i \Delta s_{t-i} + \sum_{i=1}^{n} \phi_i \Delta s^*_{t-i} \tag{19}$$

The first term on the right hand side is the error correction term. The result of estimating this equation is presented in figure 8 where we have set n=4.

We find that the error correction coefficient (μ) is very low. This suggests that the mean reversion towards the equilibrium exchange rate takes a very long time. In particular, only 0.5% of the adjustment takes place each period. It should be noted that in the simulations we have assumed a speed of adjustment in the goods market equal to 0.2. This implies that in each period the adjustment in the goods market is 20%. Thus the nominal exchange rate is considerably slower to adjust towards its equilibrium than what is implied by the speed of adjustment in the goods market. This slow adjustment of the nominal exchange rate is due to the chartist extrapolation behaviour. This phenomenon has been observed in reality. Chen et al. (2002) have recently discovered that most of the slow mean reversion of the real exchange rate is due to slow adjustment of the nominal exchange rate and not of the goods prices.

From figure 8, we also note that the changes in fundamentals have a small and insignificant impact on the change in the exchange rate. In contrast, the past changes in the exchange rate play a significant role in explaining the change in the exchange rate. These results are consistent with the empirical findings using the VAR approach, which suggests that the exchange rate is driven by its own past (see De Boeck 2000).

Thus, our model generates an empirical regularity (the ‘disconnect' puzzle) that has also been observed in reality. We can summarize the features of this puzzle as follows. First, over the very long run the exchange rate and its fundamentals are cointegrated. However, the speed with which the exchange rate reverts to its equilibrium value is very slow. Second, in the short run the exchange rate and its fundamentals are “disconnected”, i.e. they do not appear to be cointegrated. Our model closely mimics these empirical regularities.

This feature is important to analyze the effectiveness of intervention policies. It is precisely because the exchange rate is systematically disconnected from its fundamentals that foreign exchange market interventions of the sterlized type have the potential to affect the exchange rate. The scope for such interventions in the traditional rational expectations models is limited because these models predict that the exchange rate must always reflect its fundamentals.

8.2. The “excess volatility” puzzle

In this section we discuss another important empirical regularity, which has been called the “excess volatility” puzzle, i.e. the volatility of the exchange rate by far exceeds the volatility of the underlying economic variables. Baxter and Stockman (1989) and Flood and Rose (1995) found that while the movements from fixed to flexible exchange rates led to a dramatic increase in the volatility of the exchange rate no such increase could be detected in the volatility of the underlying economic variables. This contradicted the ‘news' models that predicted that the volatility of the exchange rate can only increase when the variability of the underlying fundamental variables increases.

In order to deal with this puzzle we compute the noise to signal ratio in the simulated exchange rate. We derive this noise to signal ratio as follows:

$$var(s) = var(f) + var(n) \tag{20}$$

where $var(s)$ is the variance of the simulated exchange rate, $var(f)$ is the variance of the fundamental and $var(n)$ is the residual variance (noise) produced by the non-linear speculative dynamics which is uncorrelated with $var(f)$. Rewriting (20) we obtain

$$\frac{var(n)}{var(f)} = \frac{var(s)}{var(f)} - 1 \tag{21}$$

The ratio $var(n)/var(f)$ can be interpreted as the noise to signal ratio. It gives a measure of how large the noise produced by the non-linear dynamics is with respect to the exogenous volatility of the fundamental exchange rate. We simulate this noise to signal ratio for different values of the extrapolation parameter beta (see figure 9). In addition, since this ratio is sensitive to the time interval over which it is computed we checked how it changes depending on the length of the time interval. In particular, we expect that the noise-to-signal ratio is larger when it is computed on a short than on a long time horizon. We show the results in figure 10.

First, we find that with increasing beta the noise to signal ratio increases. This implies that when the chartists increase the degree with which they extrapolate the past exchange rate movements the noise, i.e. the volatility in the exchange rate, which is unrelated to fundamentals, increases. Thus, the signal about the fundamentals that we can extract from the exchange rate becomes more clouded when the chartists extrapolate more. Second, we find that when the time horizon increases the noise-to-signal ratio declines. This is so because over long time horizons most of the volatility of the exchange rate is due to the fundamentals' volatility and very little to the endogenous noise. In contrast, over short time horizons the endogenous volatility is predominant and the signal that comes from the fundamentals is weak. This is consistent with the empirical finding concerning misalignments we discussed before.

One of the issues we will analyze when we incorporate foreign exchange market interventions into the model is whether these interventions are capable of reducing the noise to signal ratio, thereby making the exchange rate movements less sensitive to non-fundamentally driven noise.

8.3. Fat tails

It is well known that the exchange rate changes do not follow a normal distribution. Instead it has been observed that the distribution of exchange rate changes has more density around the mean than the normal and exhibits fatter tails than the normal (see de Vries 2001). This phenomenon was first discovered by Mandelbrot (1963), in commodity markets. Since then, fat tails and excess kurtosis have been discovered in many other asset markets including the exchange market. In particular, in the latter the returns have a kurtosis typically exceeding 3 and a measure of fat tails (Hill index) ranging between 2 and 5 (see Koedijk, Stork and de Vries 1992, Huisman, et al. 2002). It implies that most of the time the exchange rate movements are relatively small but that occasionally periods of turbulence occur with relatively large exchange rate changes. However, it has also been detected that the kurtosis is reduced under time aggregation. This phenomenon has been observed for most exchange rates (Lux 1998, Calvet and Fisher 2002). We checked whether this is also the case with the simulated exchange rate changes in our model.

The model was simulated using normally distributed random disturbances (with mean = 0 and standard deviation = 1). We computed the kurtosis and the Hill index of the simulated exchange rate returns. We computed the Hill index for 4 different samples of 2000 observations. In addition, we considered three different cut-off points of the tails (2.5%, 5%, 10%). We show the results of the kurtosis and of the Hill index in figure 11. We find that for a broad range of parameter values the kurtosis exceeds 3 and the Hill index indicates the presence of fat tails. Finally we check if the kurtosis of our simulated exchange rate returns declines under time aggregation. In order to do so, we chose different time aggregation periods and we computed the kurtosis of the time-aggregated exchange rate returns. We found that the kurtosis declines under time aggregation. In figure 12 we show the results for some sets of parameter values.[11] This suggests that the non-linear dynamics of the model transforms normally distributed noise in the exchange rate into exchange rate movements with tails that are significantly fatter than the normal distribution and with more density around the mean. Thus our model mimics an important empirical regularity, i.e. that exchange rate movements are characterized by tranquil periods (occurring most of the time) and turbulent periods (occurring infrequently).[12] The practical implication of the existence of fat tails is that the exchange rate dynamics is characterized by periods of tranquility and turbulence. These alternate in an unpredictable fashion. The issue is whether foreign exchange market interventions can affect this dynamic feature.

9. Is chartism evolutionary stable?

An important issue is whether chartism survives in our model. Put differently, we ask the question under which conditions chartism is profitable such that it does not disappear. It should be noted that there is a broad literature that shows that technical analysis is used widely, also by large players (see Taylor and Allen 1992, Wei and Kim 1997).

We investigate this issue by analysing how chartism evolves under different conditions. In figure 13 we show the share of chartists in the market for increasing values of the extrapolation parameter β. We obtained the chartists' weights by simulating the model over 10000 periods and computing the weight of the last period. Our first finding is that chartism does not disappear, i.e. in all simulations, for many different parameter configurations, we find that the weight attached to chartists never goes to zero. Second, for a wide range of parameter values we find that the chartists' weight in the market fluctuates around a market share, which exceeds 50%. Thus, in all our simulations of the model we find that chartist rules tend to dominate the fundamentalist rules. Third, as the extrapolation parameter β increases the weight of chartists first increases slightly. When β reaches a critical point around 0.9 (which is also the point where the dynamics switches into chaos), the market share of chartists jumps up to approximately 2/3. It then moves down to settle around 64%. This suggests that as chartists become

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11 Another empirical regularity of the distribution of exchange returns is its symmetry. We computed the skewness, and we could not reject that the distribution is symmetric.

12 The model mimicks also another important empirical regularity, i.e. the GARCH structure in the returns. We found that a GARCH (1,1) fits the simulated returns of our model.

more aggressive in extrapolating past movements of the exchange rate, the chartist rule becomes more profitable attracting more agents to become chartists. Too much extrapolation in turn reduces the relative profitability of chartism and reduces the market share of chartism. The latter result suggests that when chartists extrapolate too much the ensuing deviations from the fundamental exchange rate makes fundamentalist forecasting rules more profitable again.

A final step in the analysis consisted of analyzing the question of how the profits of the chartists and the fundamentalists evolve when the share of the chartists increases exogenously. In order to answer this question we simulated a different version of the model assuming that the shares of chartists vary exogenously from 0 to 0.9 (instead of being determined endogenously by relative profitability). We then computed the average profits of chartists' rules as the share of the chartists increases. We did the same for the fundamentalists' profits. We show the results in figures 14 and 15. The results are quite striking. As the weight of chartists increases the profits they make increases strongly. In fact the increase becomes exponential as the share of the chartists in the market becomes very high. Conversely, the profitability of fundamentalists' rules is very high when there are few chartists in the market, but drops precipituously as the chartists make an inroad. This suggests that there is a self-fulfilling evolutionary dynamics present in the system which can be described as follows. When there are no chartists in the market, fundamentalist forecasting is very profitable. However, these fundamentalists' profits are very vulnerable to an invasion by chartists. An invasion by 10% leads to a collapse of the fundamentalists' profits. From a market share of 20% on the chartists' rule becomes more profitable than the fundamentalists' rule. As the chartists increase in numbers their profits tend to increase exponentially. The reason is that the increasing importance of chartists increases the noise in the exchange rate movements, making chartism more profitable. At the same time, the chartists have the effect of "creating smoke around the fundamentals", making fundamentalists' forecasting riskier. Another way to interpret this result is that chartism creates noisy information that becomes the source of profitable speculation. The more chartists there are the more such information is created and the more profitable chartists' forecasting becomes. Thus, chartists create an informational environment which makes it rational to use chartists' rules.

Why doesn't all this not lead to a corner solution, i.e. a situation in which chartism drives out all fundamentalists? The reason has to do with risk. When the weight of chartists increases in the market, so does volatility. We show this in figure 16 where we present the exchange rate as a function of the weight of chartists. We observe that an increase in the chartists' weight leads to a significant increase in volatility especially when the weight approaches 1. Thus, as the weight of chartists in the market increases both profitability and risk of using chartist rules increase. The increasing risk is strong enough to prevent the chartists from completely driving out the fundamentalists and taking over the market.

10. The model with intervention

In this section we analyze how the exchange rate is determined when we introduce intervention by the monetary authorities in the foreign exchange rate market. From 7, it can be seen that the exchange rate is affected by the stock of net foreign assets X_t. Intervention affects the exchange rate through the changes in X_t. In particular, when the monetary authorities sell (buy) foreign exchange the stock of net foreign assets increases (decreases) thereby leading to a portfolio effect on the exchange rate. This creates the channel through which the authorities can affect the exchange rate. We model how the monetary authorities conduct this intervention. We start from the loss function of the authorities which we specify as follows.

$$L_t = (s_t - s_t^*)^2 + \omega (s_t - s_{t-1})^2 \tag{22}$$

The first term of equation 22 is the squared deviation of the exchange rate from its target level. We assume that the monetary authorities set the target equal to the fundamental rate. The second term represents the squared changes of the exchange rate, i.e. the volatility of the exchange rate. Thus, the monetary authorities also aim at minimizing the volatility of the exchange rate irrespective of the fundamental rate. The loss function therefore combines target intervention (first term) and "leaning against the wind" intervention (second term). The weight attached to the "leaning against the wind" intervention is given by ω. Since we consider only sterilized intervention, i.e. interventions that do not affect the fundamentals, the loss function does not include other possible targets that the authorities might pursue, e.g. inflation and output targets.

We can write the effect of the intervention on the exchange rate as follows:

$$s_t - s_t^* = \varphi_t (inv_t) \tag{23}$$

where $inv_t = \Delta X_t$ is the amount of intervention. Thus the monetary authorities affect the exchange rate by manipulating X_t. When $inv_t > 0$ the monetary authorities sell foreign assets, when $inv_t < 0$ they buy. φ_t represents the effect of the interventions on the exchange rate and it is determined by the nonlinear transmission dynamics of the model. From the previous section we know that this transmission process is highly uncertain. It is also time dependent, i.e. the non-linear nature of the model ensures that the same sale (purchase) of foreign exchange has a different effect on the exchange rate depending on the exact timing of the intervention. We return to this time dependent transmission effect later.

Substituting equation 23 into equation 22 and minimizing the loss function with respect to inv_t yields the optimal amount of intervention:

$$inv_t^{opt} = -\frac{1}{(1+\omega)\varphi_t} (s_{t-1} - s_t^*) \tag{24}$$

Given the uncertainty surrounding the transmission φ_t we further assume that the authorities take a conservative view, in the sense that they gradually adjust their optimal intervention policies. We represent this as follows:

$$\Delta inv_t = -\lambda \left(inv_{t-1} - inv_t^{opt} \right) \tag{25}$$

where $0 < \lambda < 1$ represents the speed with which they adjust their interventions to the optimal one.

Substituting equation 25 in equation 24 we obtain after rearranging:

$$\Delta X_t = (1-\lambda)\Delta X_{t-1} - \lambda \left(\frac{1}{(1+\omega)\,\varphi_t} \right) (s_{t-1} - s_t^*) \tag{26}$$

It can be easily seen that this expression implies that the authorities intervene in the foreign exchange rate market based on a weighted average of present and past deviations of the exchange rate from its fundamental value.[13]

We further assume that the fundamentalists take into account the optimal intervention rule of the authorities in their expectations. As a result, the fundamentalists' forecasting rule can be rewritten as:

$$E_t^f \left(\Delta s_{t+1} \right) = -\psi\lambda \left(\frac{1}{(1+\omega)\,\varphi_t} \right) (s_{t-1} - s_t^*) \left(s_t - s_t^* \right)^2 \tag{27}$$

This implies that the mean reverting process that the fundamentalists have in their forecasting rule is reinforced by the intervention.

Note that we continue to assume that the chartists use only the information about past exchange rates to forecast the future. The fact that the monetary authorities intervene in the market does not change this assumption. One could criticize this and argue that the information about intervention is readily available. This criticism, however, holds for almost all fundamental information. Most of the information about fundamentals is readily available. For example, data on inflation rates and output growth are published regularly and can be collected at (close to) zero cost. The reason why some agents decide not to use that information has to do with the fact that they do now know (or do not find it worthwile to investigate) what the effect is of changes in these fundamental variables on the exchange rate. For the same reason they do not use information on intervention activity because they find it difficult to evaluate the effect of official interventions on the exchange rate. It will turn out, however, that intervention affects the behaviour of the chartists in an indirect way, i.e. it changes the time pattern of the exchange rate movements, and thus the information set used by chartists.

13 This can be seen by writing ΔX_{t-1} as a function of. ΔX_{t-2} and $s_{t-1} - s_t^*$ as in equation and then substituting. Repeating this procedure then leads to a weighted average of past misalignments.

10.1. Results of the model with intervention

In this section we present the results of the model when the monetary authorities intervene in the market. In order to do so we set a value of $\omega = 2$, which implies that the authorities attach a relatively large weight to the volatility term in the loss function. We have also estimated φ_t by simulating the effect of changes in the net foreign assets on the exchange rate. We applied the same shock in X_t in two hundred consecutive time periods. We show the distribution in figure 17. We then computed the mean effect, which is equal to approximately 0.2 for a shock of 1 unit in X_t.

It is striking to find that the effects of the same shock in X_t but applied in different periods are so different. It is important to note that this feature is related to the sensitivity to initial conditions of the model. It implies that the transmission of the shock is time dependent and highly uncertain. This result suggests that occasional interventions are unlikely to be effective. An occasional purchase or sale by the authorities has very unpredicable effects on the exchange rate. If intervention is to have effects, it should be systematic, following a rule that is well-known in the market. The next step in the analysis consisted of simulating the model using the optimal intervention rule derived in the previous section and the mean transmission effect φ_t. In figure 18 we present some results in the form of a bifurcation diagram that relates the equilibrium exchange rate to different values of β.

This figure should be compared with figure 2 which presents the bifurcation diagram in a pure floating exchange rate regime. The comparison leads to the following conclusions. First, in the non-chaotic domain, i.e. for values of beta <0.98 the complexity of the solutions declines. More specifically, we observe that the fractured nature of the fixed-point solutions disappears. This also means that the sensitivity to initial conditions disappears. We show this in figure 19 which presents the simulated exchange rates in the time domain. The two time series of the exchange rate only differ by a small initial shock of 0.1. Contrary to the floating exchange rate regime the slightly different initial conditions do not lead to divergent exchange rate movements when the authorities apply their optimal intervention rule.

Second, the chaotic domain is displaced to the right. In other words, intervention increases the range of parameters in which we obtain fixed-point solutions. Third, in the chaotic domain the volatility of the exchange rate is dramatically reduced. Next we analyzed how intervention affects the market structure, i.e. the share of fundamentalists and chartists in the market. We found that, as a result of intervention, the share of chartists in the market declines compared to the pure floating. We present this result in figure 20 where we plotted the chartists' weight for different values of beta. Figure 20 should be compared with figure 13 that represents the chartists' weight in a floating exchange rate environment.

These results suggest that by reducing the volatility of the exchange rate, foreign exchange market interventions reduce the profitability of the chartists' rule. Thus, the chartists' share in the market declines, which implies that the complexity of the exchange rate dynamics is also reduced.

A further result relates to the noise-to-signal ratio. As in the case of pure floating we computed the noise-to-signal ratio when the authorities intervene in the foreign exchange market. Figure 21 shows the results and it should be compared with figure 10. We observe that in both pure floating and intervention, the noise-to-signal ratio declines with the time horizon. However in the case of intervention the ratio drops dramatically. This ratio now ranges between 0.07 and 0.01. The implication is that by intervening in the market the authorities are able to eliminate most of the noise produced by chartists.

From the previous analysis the following preliminary conclusion can be drawn. In a market where the interplay of chartists and fundamentalists creates a dynamics in which the exchange rate is disconnected from its fundamentals most of the time and in which the exchange rate movements are characterized by noise unrelated to fundamentals, there is scope for sterilized intervention in the foreign exchange markets. Thus even though the intervention does not affect the fundamentals, it affects the structure of the market by making noise trading less profitable. In so doing, it strengthens the role of fundamentals and tightens the link between the exchange rate and the fundamentals. The condition for intervention to have these effects is that it should be systematic and performed according to a rule that the market (the fundamentalists) understand, whereby the target that the authorities pursue is common knowledge.

10.2. Effect of shocks in the fundamentals

Sterilized intervention, if successful, affects the structure of the foreign exchange market changing the composition of chartists' and fundamentalists' forecasting activities. It also affects the structure of the economy in another way. In particular it affects the transmission mechanism of shocks in the fundamentals on the exchange rate. We show this by performing simulations whereby we introduce a permanent shock in the fundamental and analyze the effect on the exchange rate both in a situation of floating and of intervention. We show the results for a particular parameter configuration in figures 22 and 23. The figures are constructed in the same way as figure 5. The results are striking. In the case of floating we find the same result as the one discussed earlier, i.e. the shock in the fundamental has very unpredictable effects on the exchange rate (note that there is no stochastics in the model when this shock is performed). With intervention we find that the same shock quickly leads to a perfectly predictable effect on the exchange rate. There is no complexity anymore. This result is related to the wider effect of intervention in that it reduces the chartists' driven noise and the complex non-linearities that follow from the interaction of chartists and fundamentalists.

11. Is intervention sustainable?

An important issue relates to the sustainability of intervention policies. Put differently it is the question of whether the monetary authorities hold enough international reserves to use the kind of optimal intervention rules analyzed in the previous sections. In order to analyze this issue we studied the behaviour of the stock of international reserves over time. We show an example in figure 24 (beta=0.8, theta=0.2, omega=1.1, elast=0.8), and we start with an intitial stock of 1000. We did this for many other parameter configurations. A minimal condition for sustainability is that the stock of international reserves should be mean-reverting, i.e. the time series should not exhibit a unit root. If it does, we know that over time the stock of international reserves will be depleted or the country will own all the reserves of the world. We therefore performed an augmented Dickey-Fuller test on the simulated series of international reserves, with different numbers of lags. The results allow us to reject the hypothesis of a unit root.[14] This implies that there is a mean reverting process. Thus, the optimal intervention rule passes the necessary test of sutainability. This, of course, is not sufficient. It could still be that at some point the authorities face a constraint because the stock has dropped below a certain limit, even though this drop is temporary.

12. On the nature optimal intervention rules

In the previous section we showed that some (sterilized) intervention rules are capable of reducing the noise in the foreign exchange market and of keeping the exchange rate closer to its fundamental value than the free float regime does. In this section we analyze the nature of these successful rules further. We highlight two features of these rules. First, there should be some anchoring of the market's expectations. Second the rules should be applied gradually.

The importance of anchoring can be shown as follows. We analyzed what would happen if the authorities would apply a pure "leaning against the wind" strategy. This means setting $\omega = 1$ in equation 22. In this case the authorities do not try to move the exchange rate towards its fundamental value, but are only concerned with reducing short-term volatility. We show a simulation result in figure 25, in the form of a bifurcation diagram like figure 18. It can be seen that this intervention rule does little to reduce the complexity of the exchange rate dynamics. Our interpretation is that a pure leaning agains the wind strategy introduces a new time pattern of the exchange rates in which chartists can build their forecasting rules, thereby amplifying the actions of the monetary authorities. The mean reverting process is weakened compared to the intervention rules in which the authorities pursue a particular target for the exchange rate. Optimal intervention rules should also be applied in a gradual way. We experimented with intervention rules in which we eliminated gradualism by setting λ=1 in equation 26. We found that such a rule quickly destabilizes the foreign exchange market. Thus, too much ambition of the monetary authorities aiming at quickly guiding the exchange rate to its fundamental rate should not be used as it amplifies the volatility of the exchange rate compared to a free float regime.

[14] ADF with a number of lags equal to three is -32.55. The autoregressive parameter on the stock of international reserves is 0.82.

13. Conclusion

In traditional exchange rate models in which the exchange rate always reflects present and future expected fundamentals, there is very little scope for interventions in the foreign exchange market. In these models interventions can be effective in only two cases. One is that the monetary authorities are willing to change their domestic policies thereby changing the time path of fundamentals. The second case is that the interventions signal a future change in policies. This analysis has very much conditioned academic thinking on the subject leading to the view that sterilized interventions are an exercise in futility. The fact, however, is that most countries, and in particular Asian countries, which manage their exchange rates, use sterilized interventions and usually do not signal any future change of policies when they intervene in the market. Are these countries then acting foolishly?

We argued in this paper that one needs other than rational-expectations-representative agents models to analyze interventions in the foreign exchange market. We developed a model with heterogeneity of beliefs about the future exchange rate. In such a model the exchange rate deviates from its fundamental rate most of the time. In addition, noise is created endogenously by the interaction of chartists (noise traders) and fundamentalists without any connection with the stochastic shocks in the fundamentals. Finally in this model periods of tranquility and turbulence alternate unrelated to turbulence in the fundamentals. We showed that in the context of such a model sterilized intervention can be quite effective in reducing the endogenous noise and in bringing the exchange rate closer to its fundamentals. This is achieved without this intervention signalling anything about the future fundamentals. The reason why this intervention can be quite effective is that it tends to reduce the noise generated by chartists' forecasting rules. As a result, it also reduces the profitability of chartists' forecasting rules and tends to drive out chartists from the market.

We also indentified some conditions for this intervention to be effective. The first one is that it should be systematic. We found that occasional interventions do not work well, because the effect on the exchange rate of these occasional purchases and sales of foreign exchange are very unpredictable. Second, the authorities have to be transparent about the intervention rule they follow, in particular about the target exchange rate they pursue. This makes it possible for the fundamentalists to incorporate this rule in their expectations, thereby enhancing its effects. Third, pure leaning against the wind rules do not work well. The authorities need to pursue a target for the exchange rate. This has the effect of anchoring the expectations of the fundamentalists in the market, thereby increasing mean reversion in the exchange rate dynamics. Finally, the intervention rules should be applied gradually. When authorities intervene so as to bring the exchange rate too quickly towards its fundamental, they may actually destabilize the market.

There are several issues that were not solved in this paper. The first one has to do with the sustainability of the intervention rules analyzed this paper. We found that the stock of international reserves shows mean reversion when the authorities apply intervention. However, sufficiently large shocks may still lead to losses in international reserves that are large enough to make the interventions unsustainable. This possibility leads to the suggestion that these intervention rules would gain much in credibility when applied in a multilateral context.

The second problem has to do with the choice of the target exchange rate. We assumed that the authorities set the target exchange rate equal to the fundamental rate. Problems could arise if the authorities fail to select the right target exchange rate. We intend to pursue this matter in our future research.

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Figure 1

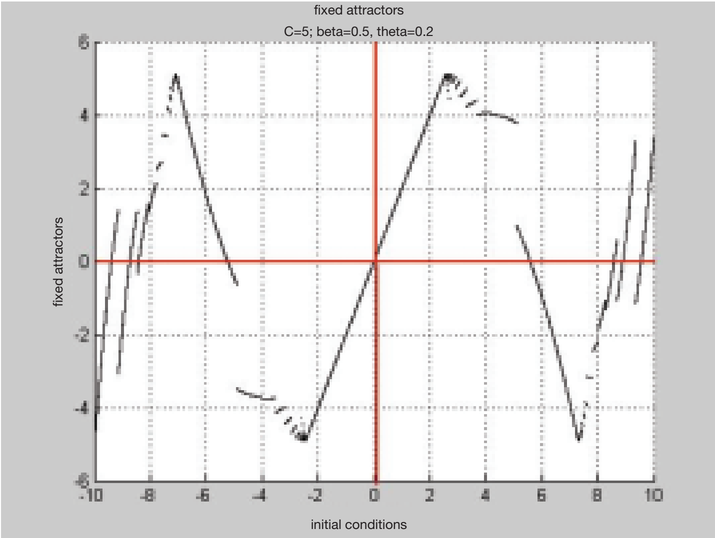
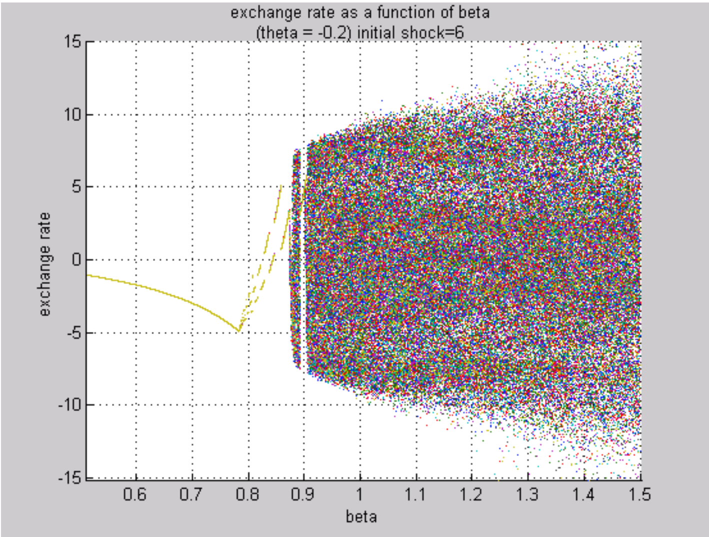


Figure 2



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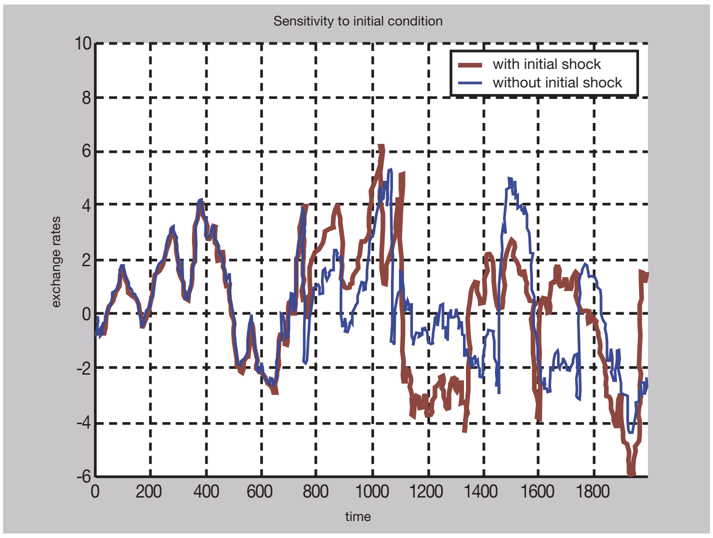

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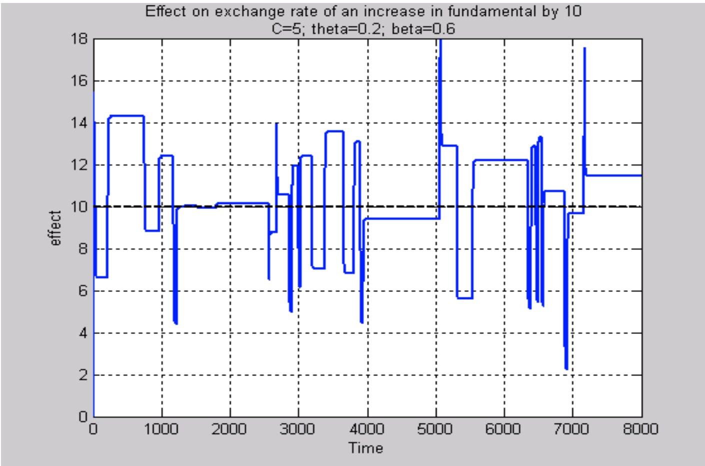

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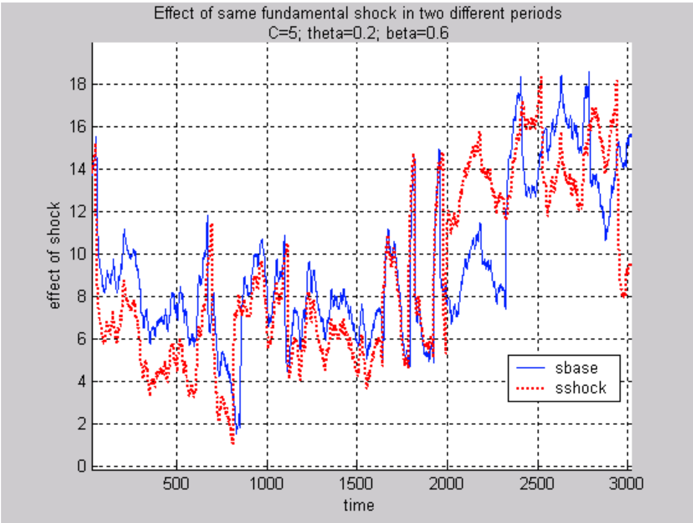


Figure 6. C=5; beta=0.5; theta=0.2

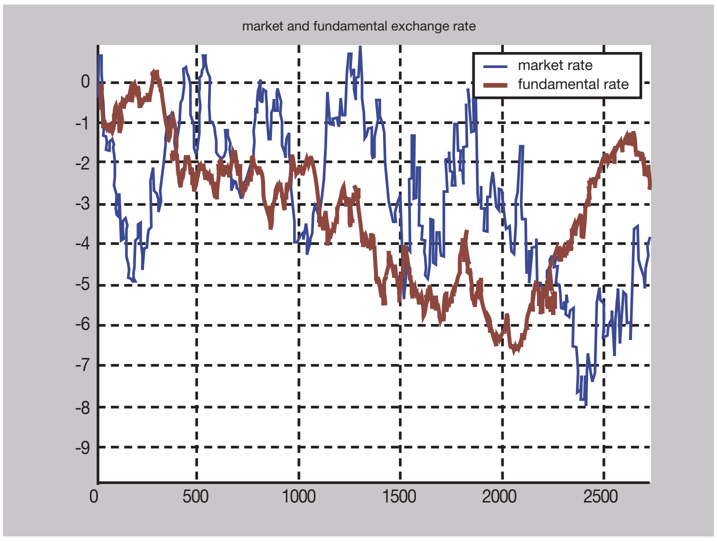


Figure 7. Parameter estimates of EC model (equation 19)

Error Correction Term		Δs_{t-i}				Δs^*_{t-i}			
μ	γ	λ_1	λ_2	λ_3	λ_4	φ_1	φ_2	φ_3	φ_4
-0.005 -6.4	1.06 11.3	0.18 16.4	0.15 13.1	0.09 7.06	0.05 4.3	0.04 2.2	0.03 1.6	0.02 1.3	0.01 1.06

Figure 8

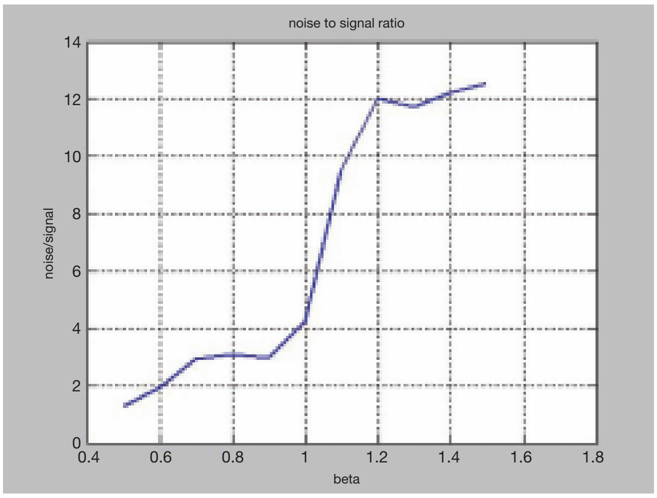


Figure 9. C=5; beta=0.5; theta=0.2; the noise to signal ratios are averages of 20 simulations.

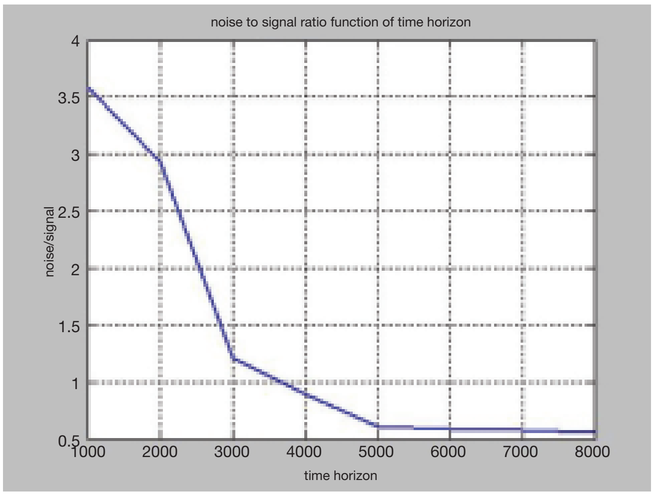


Figure 10

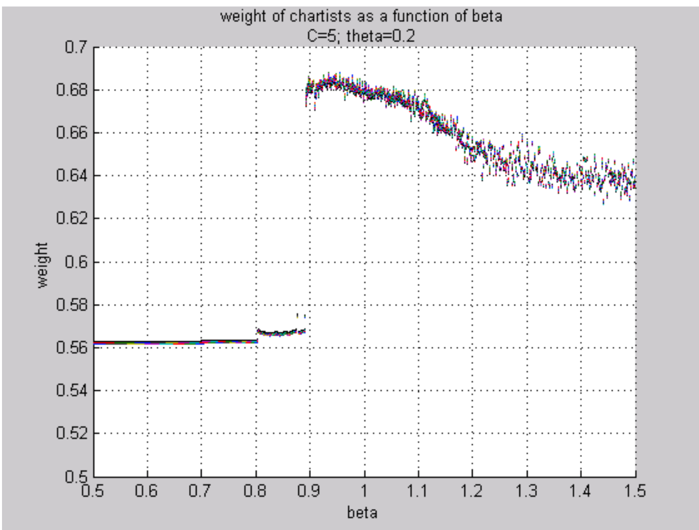
		Median Hill index (4 samples 2000 observations)		
Parameter values	Kurtosis	2.5% tail	5% tail	10% tail
c=5, beta=0.5, theta=0.20	185.4	2.1	2.8	3.1
c=5, beta=0.7, theta=0.20	66.7	1.7	1.6	1.9
c=5, beta=0.7, theta=0.05	5.9	4.6	4.3	4.0
c=5, beta=0.5, theta=0.10	48.8	2.9	3.4	3.4
c=5, beta=0.5, theta=0.05	8.5	4.2	4.2	4.0

Figure 11

Kurtosis under time-aggregation

Parameter values	1 period returns	5 periods returns	10 periods returns	25 periods returns
C=5, theta=0.2, beta=1.0	7.1	4.5	4.0	2.2
C=5, theta=0.2, beta=0.8	40.3	13.3	10.3	4.6
C=5, theta=0.2, beta=0.5	182.9	37.9	24.2	9.9
C=5, theta=0.1, beta=1.0	3.8	3.2	3.3	2.4
C=5, theta=0.3, beta=1.0	8.9	4.9	3.6	2.2

Figure 12



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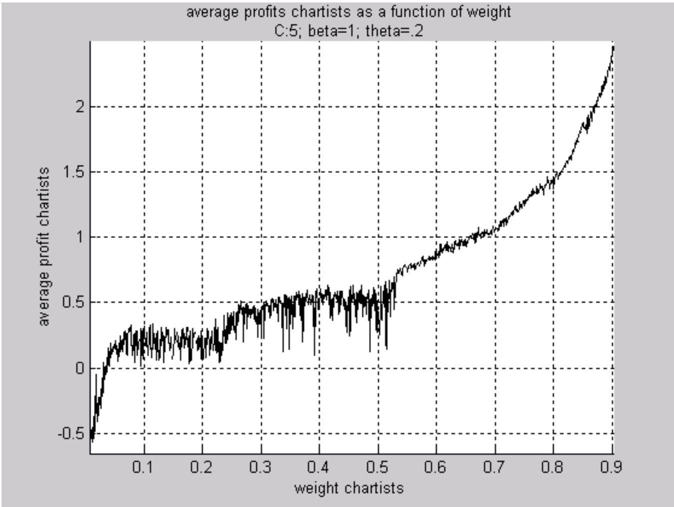

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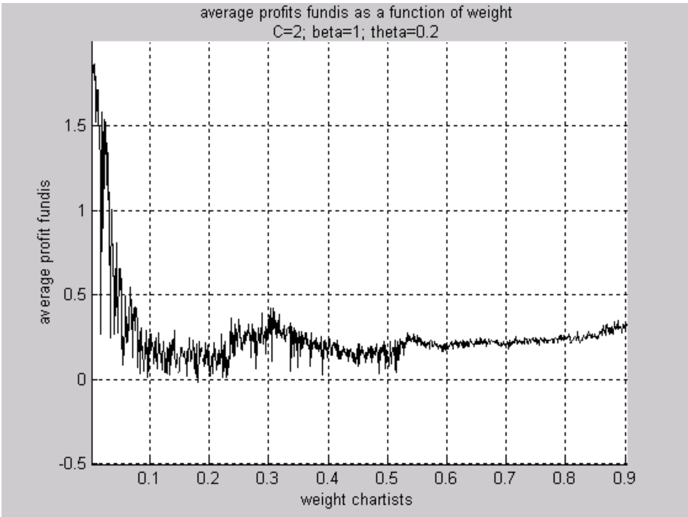

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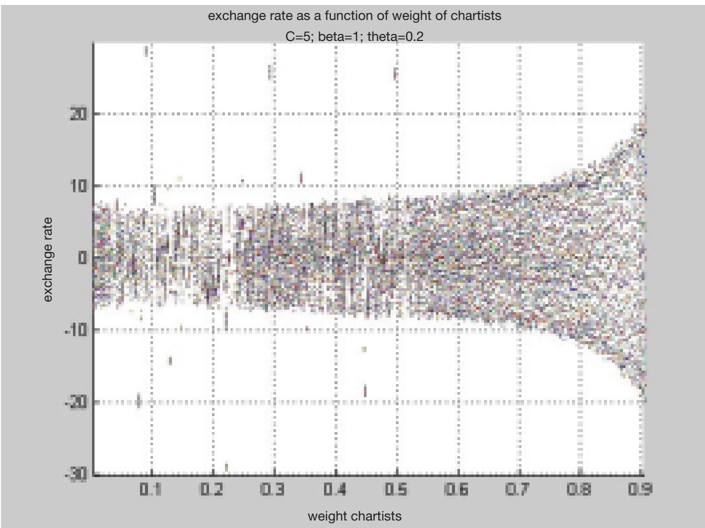

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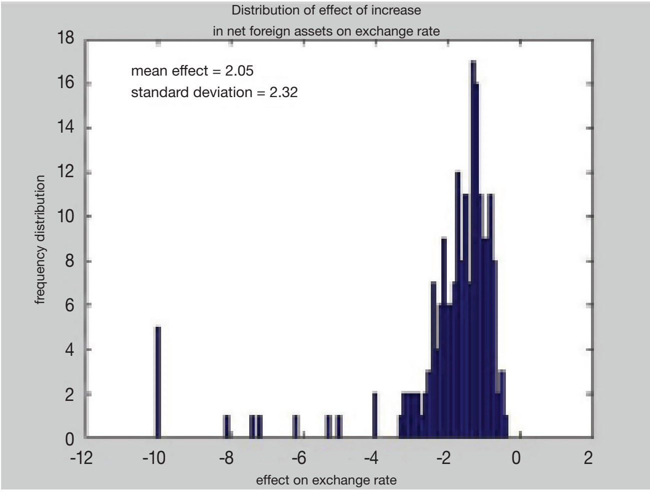

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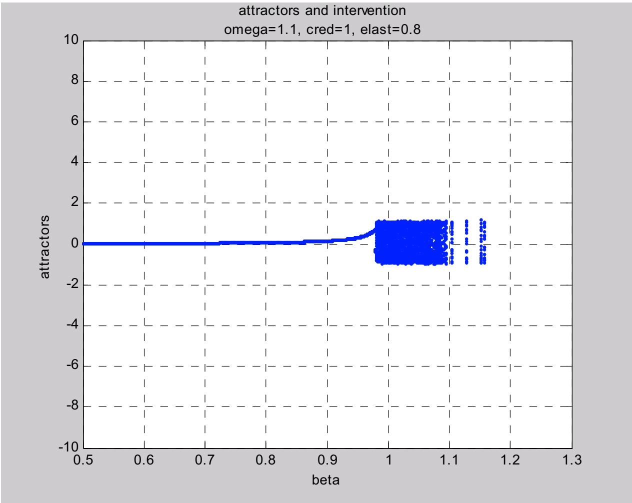

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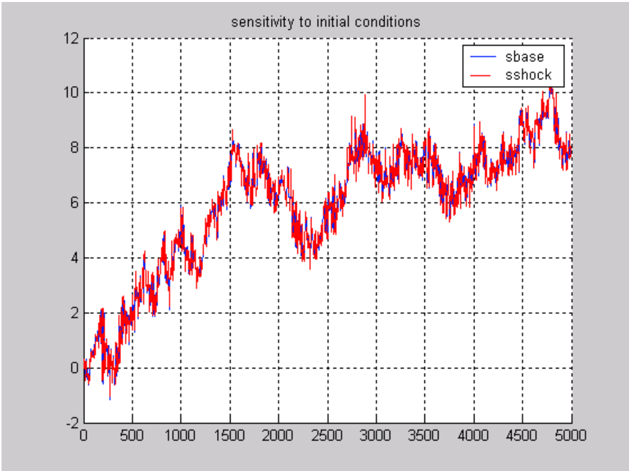

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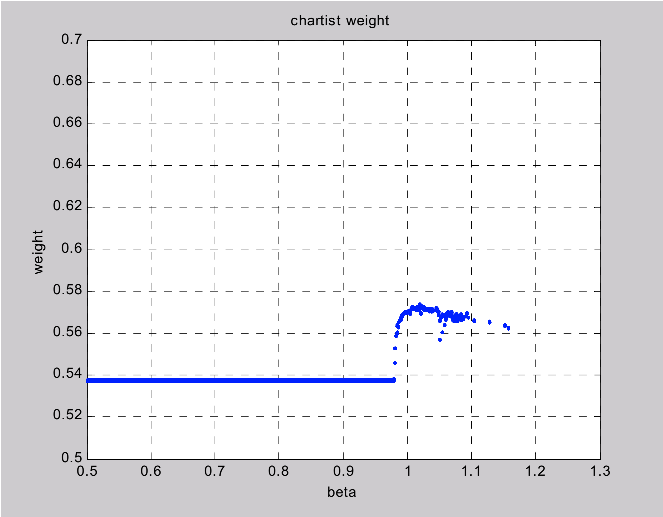

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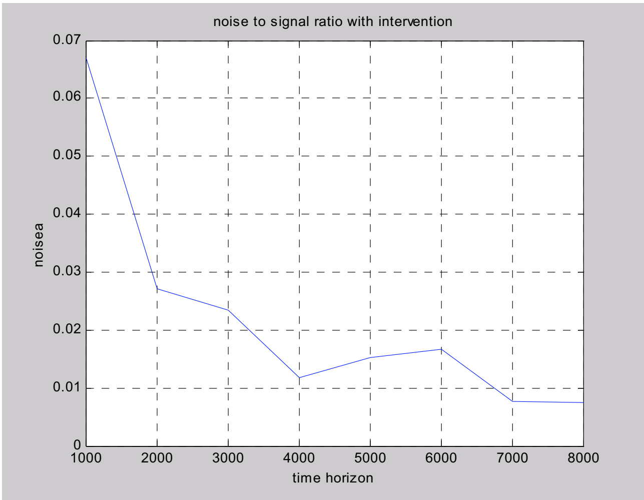

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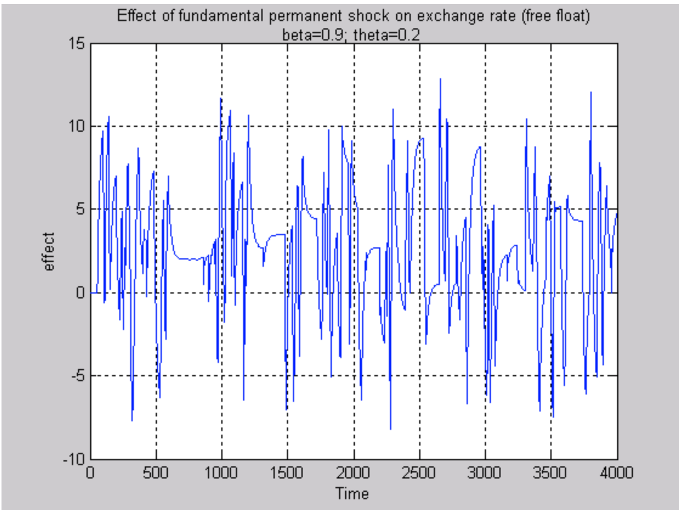

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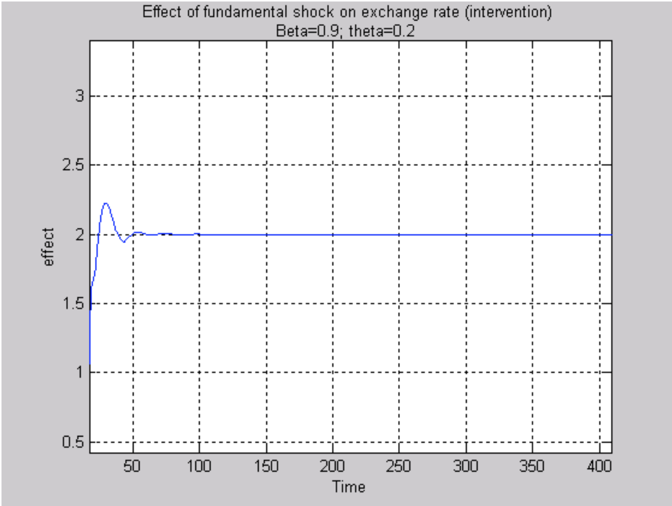

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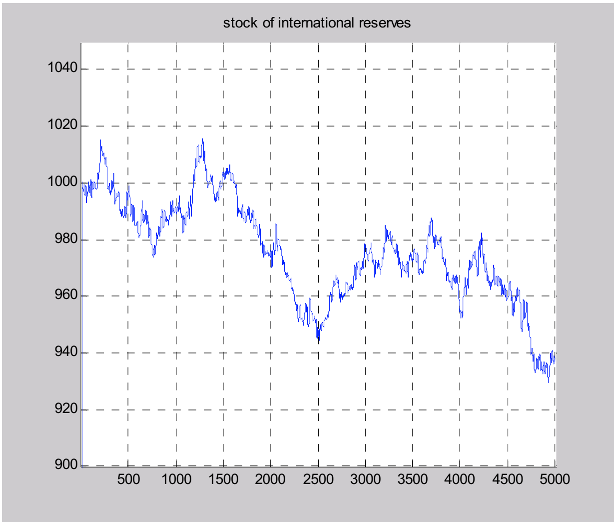

Figure 24