HONG KONG INSTITUTE FOR MONETARY RESEARCH

TERMS-OF-TRADE CHANGES, REAL GDP, AND REAL VALUE ADDED IN THE OPEN ECONOMY: REASSESSING HONG KONG'S GROWTH PERFORMANCE

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Terms-of-Trade Changes, Real GDP, and Real Value Added in the Open Economy:
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March 2006

Abstract

Real GDP tends to underestimate the increase in real domestic value added, real GDI, and welfare when the terms of trade improve. An improvement in the terms of trade is similar to a technological progress. The national accounts treat the two phenomena very unevenly, however, with a change in the terms of trade considered to be a price event and thus not incorporated in the computation of real GDP. The impact of a change in the real exchange rate on real value added is not taken into account by real GDP either. Given its extreme openness and in view of the massive terms-of-trade improvements that it has enjoyed over the past forty years, Hong Kong makes for an interesting case study. Our findings suggest that average real growth has been underestimated by real GDP by approximated 0.4% per annum between 1961 and 2003. This adds up to a gap of close to 20% of GDP over the entire period.

Key Words: Terms of trade, real exchange rate, real GDP, real GDI, real value added, technological change, total factor productivity, index numbers
JEL classification: O11, O41, C43, F11

1. Introduction

Over the past four decades, Hong Kong's terms of trade have improved massively. As shown by Figure 1, the terms of trade, measured by the price of exports relative to the price of imports, have increased by nearly 50% between 1961 and 2003. Much of the improvement occurred during the sixties and the seventies. A bettering of the terms of trade amounts to a windfall gain for the country as a whole and it implies without any doubt an increase in its real value added, its real income, and its economic welfare. An improvement in the terms of trade essentially means that the country gets more for less. This phenomenon is similar to a technological progress.[1] Contrary to a technological progress, however, a change in the terms of trade is treated by the national accounts as a price phenomenon, rather than as a real effect. Consequently, the beneficial effect of an improvement in the terms of trade is not taken into account by real gross domestic product (GDP). Real domestic value added growth will thus be underestimated in countries that experience an improvement in their terms of trade. Similarly, in international comparisons, the real income of countries that enjoy relatively favourable terms of trade will tend to be underestimated by real GDP.[2] As documented by Figure 2, Hong Kong is an exceptionally open economy, with the GDP shares of imports and exports in excess of 1.6 in recent years. In view of this extreme openness, one can suspect that real GDP as it is conventionally measured has underestimated the growth in real domestic value added over the years, particularly so during the sixties and seventies. This paper examines this question in more detail. It essentially adopts the approach proposed by Kohli (2004a), but it innovates by using a definition of the real exchange rate that treats imports and exports symmetrically. Consequently the decomposition of the trade gains between the terms-of-trade effect and the real-exchange-rate effect is somewhat modified.

2. Nominal vs. Real GDP

A country's *nominal* GDP measures the total value of all final goods and services produced during a given period of time. Nominal GDP can also be interpreted as the country's nominal gross domestic income (GDI) or its nominal domestic value added.[3] Nominal GDP can be measured by looking at the expenditure side. Let us assume that gross output can be disaggregated into two components, a nontraded good (N) intended for domestic use (it is an aggregate of private household consumption, investment, and government purchases), and exports (X).[4] Since some of these goods and services have foreign contents, one must subtract imports (M). Nominal GDP (π) is thus equal to:

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[1] See Diewert and Morrison (1986), Kohli (1990, 2004a), as well as Feenstra, Reinsdorf, Slaughter, and Harper (2005).

[2] See Feenstra, Heston, Timmer, and Deng (2004).

[3] Throughout the paper we use the terms *income and value added* interchangeably.

[4] In this paper (except for the simple illustration of Section 4) imports and exports are treated as middle products. Exports are thus conceptually different from goods intended for domestic use, since the former must still be processed by foreign producers before being ready to meet final demand.

$$\pi_t \equiv v_{N,t} + v_{X,t} - v_{M,t}, \tag{1}$$

where the v's denote nominal values and t is for the time period. Nominal GDP amounted to about 1,220 billion HK dollars in 2003. As a comparison, it was equal to 7 billion HK dollars in 1961.

For many purposes, it is *real* – rather than nominal – value added that is of interest. The nominal value of each GDP component can be interpreted as the product of a quantity and of an average price. Thus, for the ith GDP component:

$$v_{i,t} \equiv p_{i,t} q_{i,t}, \quad i \in \{N, X, M\}, \tag{2}$$

where p_i is the price of the ith component, and q_i is the corresponding quantity. It is standard practice to normalise prices to unity for some base period (period *0*). By dividing each component of nominal GDP by the corresponding price index and adding up the quantities thus obtained one gets *real* GDP (q) – also known as constant dollar GDP – such as it is measured in most countries, including in Hong Kong:

$$q_t \equiv \frac{v_{N,t}}{p_{N,t}} + \frac{v_{X,t}}{p_{X,t}} - \frac{v_{M,t}}{p_{M,t}} = q_{N,t} + q_{X,t} - q_{M,t}. \tag{3}$$

The real GDP index given by (3) is known in the literature as a Laspeyres quantity index. Taking again 2003 figures and using 2000 as the base period, one gets a value of real GDP of about 1,361 billion HK dollars (at 2000 prices). The corresponding figure for 1961 is 87 billion HK dollars. The path of real GDP between 1961 and 2003 is shown in Figure 3.

It is tempting to divide nominal GDP by real GDP to obtain a sort of average price of GDP, also known as the GDP deflator (p):

$$p_t \equiv \frac{\pi_t}{q_t} = \frac{v_{N,t} + v_{X,t} - v_{M,t}}{\dfrac{v_{N,t}}{p_{N,t}} + \dfrac{v_{X,t}}{p_{X,t}} - \dfrac{v_{M,t}}{p_{M,t}}} = \frac{1}{s_{N,t}\dfrac{1}{p_{N,t}} + s_{X,t}\dfrac{1}{p_{X,t}} - s_{M,t}\dfrac{1}{p_{M,t}}}, \tag{4}$$

where s_N, s_X and s_M are the GDP shares of domestic goods, exports, and imports respectively. The GDP deflator is thus a harmonic mean of the prices of the various GDP components. It is known in the economic literature as a Paasche price index.

It is common practice to use real GDP data to compute growth rates over consecutive periods. Let $Q_{t,t-1}$ be one plus the rate of growth of real GDP between period *t-1* and period *t*:

$$Q_{t,t-1} \equiv \frac{q_t}{q_{t-1}} = \frac{\sum_i \pm q_{i,t}}{\sum_i \pm q_{i,t-1}} = \frac{\sum_i \pm q_{i,t} / \sum_i \pm q_{i,0}}{\sum_i \pm q_{i,t-1} / \sum_i \pm q_{i,0}} = \frac{\sum_i \pm s_{i,0}\dfrac{q_{i,t}}{q_{i,0}}}{\sum_i \pm s_{i,0}\dfrac{q_{i,t-1}}{q_{i,0}}}, \tag{5}$$

$$i \in \{N, X, M\},$$

where the sign is negative for imports and positive for the other two components, and where we have taken into account the fact that base-period (period *0*) prices are normalized to unity. It is important to note, however, that $Q_{t,t-1}$ so defined is a ratio of two direct Laspeyres quantity indices, but it is not itself a Laspeyres quantity index, unless period *t-1* happens to be the base period. Direct indices are defined relative to a base period, but they are not well suited to make comparisons over consecutive periods not involving the base period.[5] Index $Q_{t,t-1}$ can be contrasted with a true Laspeyres quantity index $G_{t,t-1}$ of real GDP:

$$G_{t,t-1} \equiv \sum_i \pm s_{i,t-1} \frac{q_{i,t}}{q_{i,t-1}} \; , \qquad i \in \{N, X, M\} \cdot \tag{6}$$

Time series of real GDP over longer periods of time can be obtained as chained indices by compounding the individual elements. Several countries have recently switched from runs of direct Laspeyres quantity indices to chained Laspeyres quantity indices to measure real GDP in conformity with the recommendations by Eurostat in the context of the 1995 system of European Standardised Accounts (ESA95). In that case the implicit GDP price deflator has the form of a chained Paasche price index.

While the Laspeyres and Paasche indices are still today the ones most commonly used in practice, other, and better, formulas are available. One need only think of the Fisher and of the Törnqvist indices, two members of the family of *superlative* index numbers.[6] The Törnqvist index is particularly relevant for what follows. Let $P_{t,t-1}$ be the Törnqvist price index of GDP over consecutive periods. It is as follows:

$$P_{t,t-1} \equiv \exp\left[\sum_i \pm \frac{s_{i,t} + s_{i,t-1}}{2} \ln \frac{p_{i,t}}{p_{i,t-1}} \right], \quad i \in \{N, X, M\}, \tag{7}$$

where the sign is negative for imports, and positive for the two other components. Let $\Pi_{t,t-1}$ be one plus the rate of change in nominal GDP:

$$\Pi_{t,t-1} \equiv \frac{\pi_t}{\pi_{t-1}} \cdot \tag{8}$$

Deflating the nominal GDP index by $P_{t,t-1}$ yields the implicit Törnqvist index of real GDP ($Y_{t,t-1}$), a superlative quantity index:[7]

$$Y_{t,t-1} \equiv \frac{\Pi_{t,t-1}}{P_{t,t-1}} \cdot \tag{9}$$

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[5]   See Kohli (2004d).

[6]   See Diewert (1976).

[7]   See Kohli (2004b).

3. Real Gross Domestic Income

We can define real GDI (z) as nominal GDI (or GDP) divided by the price of domestic expenditures (p_N). We thus obtain:

$$z_t \equiv \frac{\pi_t}{p_{N,t}} = q_{N,t} + \frac{p_{X,t}}{p_{N,t}} q_{X,t} - \frac{p_{M,t}}{p_{N,t}} q_{M,t} = q_{N,t} + e_t h_t^{-1/2} q_{X,t} - e_t h_t^{1/2} q_{M,t} , \tag{10}$$

where h is the inverse of the terms of trade and e is an index of the price of tradables relative to the price of nontradables:

$$h_t \equiv \frac{p_{M,t}}{p_{X,t}} \tag{11}$$

$$e_t \equiv \frac{p_{X,t}^{1/2} p_{M,t}^{1/2}}{p_{N,t}} . \tag{12}$$

The numerator in (12) is a Cobb-Douglas price index of tradables, with imports and exports having equal weights.[8] For given terms of trade, a change in e can be interpreted as a change in the real exchange rate, an increase in e being equivalent to a real depreciation of the home currency. The path of the real exchange rate so defined is shown in Figure 1 as well. It can be seen that the real exchange rate was fairly steady over the first half of the sample period. The fall in the price of tradables relative to the price in nontradables during the eighties and the early nineties reveals a real appreciation of the HK dollar. Definition (12) is somewhat different from the one used in Kohli (2004a) where the real exchange rate was simply taken as the price of exports relative to the price of nontraded goods. The advantage of (12) is that both imports and exports are treated symmetrically. Comparing (10) with (3), one sees that the crucial difference between real GDI and real GDP is that in the former the quantity of exports and imports are weighted by the terms of trade and the real exchange rate. A change in the terms of trade or in the real exchange rate will have a direct impact on real GDI, but not on real GDP. We will show in Sections 5 and 6 how the real GDI effects of changes in the terms of trade and the real exchange rate can be measured.

The change in real GDI over time is captured by index ($Z_{t,t-1}$) defined as follows:

$$Z_{t,t-1} \equiv \frac{z_t}{z_{t-1}} = \frac{\Pi_{t,t-1}}{P_{N,t,t-1}} \tag{13}$$

where $P_{N,t,t-1}$ is one plus the rate of inflation in terms of the domestic good:

$$P_{N,t,t-1} \equiv \frac{p_{N,t}}{p_{N,t-1}} . \tag{14}$$

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[8]   Alternatively, the weights could be set to the average shares of imports and exports in total trade. It turns out, however, that this would make little difference in the case of Hong Kong, since trade was fairly balanced on average over the sample period.

4. Preliminary Analysis

The conceptual difference between real GDP and real GDI can be illustrated with the help of a simple model. In what follows, we will treat imports as inputs to the production process, but our analysis is also valid in the context of consumer theory, as opposed to production theory. Let us assume that production involves one domestic factor of production, an aggregate of labour and capital, as well as imported products. Let us also assume for the time being that all outputs (domestic output and exports) can be aggregated into a composite good. The country's technology can then be described by the following aggregate production function:

$$q_{Y,t} = f(q_{M,t}, v_t),$$ (15)

where q_Y is the total quantity of output, and v is the endowment of the domestic factor of production. We assume that this production function is increasing, linearly homogeneous and concave. Furthermore, we assume perfect competition, given factor endowments, and exogenous terms of trade as it is standard in international trade theory.

The production function is shown in Figure 4, with gross output as a function of the quantity of imports, for a given endowment of the domestic factor.[9] The slope of the production function can be interpreted as the marginal product of imports. Let the relative price of imports – the inverse of the terms of trade – be given by the slope of line *BC*. This slope is unity since all prices are typically normalised to one in the base period. Profit maximisation by producers will lead to an equilibrium at point *C* where the marginal product of imports is equal to their marginal cost. The volume of imports is the distance *OD* and total output is equal to *OE*. If trade is balanced, exports are equal to *BE*. The distance *OB* can be interpreted as real income, real value added, or real GDP:

$$q_t = q_{Y,t} - q_{M,t}.$$ (16)

Assume now that the terms of trade improve, as the result, for instance, of a drop in import prices. The terms of trade are now given by the slope of *B'C'*, and equilibrium moves from point *C* to point *C'*. The country imports more. The marginal product of imports admittedly falls, but their real price is now lower. Gross output is now equal to *OE'*. Economic welfare has clearly increased since real income – or real value added – went from *OB* to *OB'*. Real GDP, on the other hand, *falls* from *OB* to *OF*: point *F* is at the intersection between the vertical axis and a unit-sloped line through *C'*. In accordance with the definition of real GDP, the distance *OF* is equal to *OE'* (gross output, q_Y) minus *OD'* (imports, q_M). Thus, real GDP falls, even though real income, real value added and economic welfare unambiguously increase. This clearly illustrates the difficulties involved with this measure of a country's real value added.[10]

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[9] See Kohli (1983, 2004a) for further details.

[10] Note that there are only two states (e.g. periods) in this example. Hence our demonstration is valid independently of whether one uses direct or chained indices.

In the model of Figure 4, imports are treated as intermediate goods. This is consistent with the evidence that the bulk of world trade is in raw material and intermediate products. Moreover, even most so-called finished products are still subject to many domestic charges before they reach final demand, so that a significant proportion of their final price tag is generally accounted for by domestic value added. Nevertheless, the perverse effect of a change in the terms of trade on real GDP can also be illustrated with the conventional model of international trade theory, the Heckscher-Ohlin model, where all goods are treated as endproducts. In Figure 5, the initial terms of trade are given by the inverse of the slope of line *PC*. Production takes place at point *P* on the production possibilities frontier, whereas if trade is balanced, consumption takes place at point *C*. Next let the terms of trade improve, so that the international price line is now given by *P'C'*. Production shifts towards the north-west, from *P* to *P'*, whereas consumption moves from *C* to *C'*. *C'* is on a higher indifference curve, which clearly demonstrates the increase in welfare. Yet real GDP *falls*, from *OA* to *OA'*, *A'* being at the intersection between the vertical axis and a line through *P'*, with the same slope as *PC*.

Thus, not only is the effect of an improvement in the terms of trade on real income underestimated by real GDP, but, even more seriously, the change in real GDP goes in the wrong direction! The intuitive explanation for this phenomenon is as follows. When import prices fall, the country can afford to import more. Yet, since real GDP is obtained by subtracting imports valued at their base period prices, i.e. without taking into account the lower price of imports, one ends up subtracting too much and one thus gets a real GDP figure that is too low.

Another way to look at the problem is to consider the effect of a change in the terms of trade on the GDP deflator. A drop in the price of imports leads to an increase in the deflator (since imports enter with a negative weight), even though no price has actually increased. Incidentally, this shows that the GDP deflator is a poor index of the general price level, since the drop in the price of imports has no inflationary effects, quite the contrary. It is obvious, therefore, that if the GDP deflator overestimates the price level, real GDP will underestimate the quantity of real value added.[11]

As an analogy, think of a farmer who grows wheat in his field, using his labour and fertiliser as the only inputs (for simplicity, we are ignoring the other inputs such as seeds and capital). Assume that the price of wheat is constant, but that for some reason the price of fertiliser falls. Everyone would agree that this is very good news for the farmer whose income will increase, even if he does not change his behaviour. In fact, he will probably be tempted to increase his use of fertiliser, which has become cheaper, in order to increase his output of wheat somewhat, and thus to raise his income even more. True, using more fertiliser will increase the output of wheat by only a small amount since the marginal product of fertiliser is falling, but it would be absurd to simply subtract the quantity of fertiliser used from the quantity of wheat produced to come to the conclusion that the real value added by the farmer has fallen.

In a recent paper, Feenstra et al. (2004) draw a distinction between two measures of real GDP: an output side measure, which they denote by GDP^o, and an expenditure-side measure, which they denote by GDP^e. The former is obtained by deflating nominal GDP by a price index for output (the usual GDP

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[11] In case of a terms-of-trade deterioration, the opposite happens: the GDP price deflator underestimates the change in the price level.

deflator), while the latter is obtained by using instead a price index for expenditures (a domestic absorption price index). The distinction between GDP[o] and GDP[e] is thus essentially the same as between real GDP and real GDI.[12] Besides terminology,[13] the main differences between the two treatments are that (1) our measures are based on an explicit production model, (2) we use superlative indices rather than Paasche and Laspeyres indices, and (3) we do not correct for purchasing power differences since, by focusing on a single economy (Hong Kong), we do not need to make international comparisons. The main purpose of Feenstra et al. (2004) is precisely to produce purchasing-power-parity adjusted expenditure price deflators in order to make it possible to compare international standards of living using the Penn World Tables.

5. Generalisation

The model of the previous section was rather restrictive. Fortunately, it can easily be generalised to allow for technological change and to include many inputs and many outputs. In what follows, we will consider two domestic factors, labour (L) and capital (K), and we will again split gross output into exports (X) and domestic goods (N). Let T_t be the production possibilities set at time t. We assume that T_t is a convex cone. The aggregate technology can be described by a real GDI function defined as follows:[14]

$$z_t = z(h_t, e_t, v_{K,t}, v_{L,t}, t) \equiv \max_{q_N, q_X, q_M} \left\{ \begin{array}{l} q_{N,t} + e_t h_t^{-1/2} q_{X,t} - e_t h_t^{1/2} q_{M,t} : \\ (q_{N,t}, q_{X,t}, q_{M,t}, v_{K,t}, v_{L,t}) \in T_t \end{array} \right\}. \tag{17}$$

We show in the Appendix that the real GDI function has the following slope properties:

$$\frac{\partial z(\cdot)}{\partial h} = -\frac{1}{2} \frac{e_t}{h_t} \left(h_t^{-1/2} q_{X,t} + h_t^{1/2} q_{M,t} \right) \tag{18}$$

$$\frac{\partial z(\cdot)}{\partial e} = h_t^{-1/2} q_{X,t} - h_t^{1/2} q_{M,t} \tag{19}$$

$$\frac{\partial z(\cdot)}{\partial v_K} = \frac{w_{K,t}}{p_{N,t}} \tag{20}$$

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[12] Compare equations (1) and (2) in Feenstra et al. (2004) with our equations (9) and (13), or equations (42) and (43) in Kohli (2004a).

[13] Feenstra et al. (2004) reject the term GDI because "it is suggestive of the income-approach to measuring GDP (through adding up the earnings of factors) which we do not use". This argument is not convincing, since nominal GDP is theoretically equivalent to nominal GDI, whereas nominal domestic expenditures will not equal nominal GDP if trade is not balanced. Furthermore, nominal GDP itself can be measured in two difference ways, by adding up value added by industry, or, as it is done here, by adding up expenditure components. Finally, the distinction output/expenditures suggests that trade is an activity that takes place once that production is completed, even though, as Feenstra et al. recognise, most trade is in intermediate goods and thus takes place during the production process.

[14] The real GDI function can be viewed as a modified nominal GDP function; see Kohli (2004c).

$$\frac{\partial z(\cdot)}{\partial v_L} = \frac{w_{L,t}}{p_{N,t}} \tag{21}$$

$$\frac{\partial z(\cdot)}{\partial t} = \mu_t z_t, \tag{22}$$

where μ is the instantaneous rate of technological change.

Following Diewert and Morrison (1986), and using (17) as a starting point, we can define the following index to capture the contribution of changes in the terms of trade to real GDI:

$$Z_{H,t,t-1} \equiv \sqrt{\frac{z(h_t, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)}{z(h_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \times \frac{z(h_t, e_t, v_{K,t}, v_{L,t}, t)}{z(h_{t-1}, e_t, v_{K,t}, v_{L,t}, t)}} \,. \tag{23}$$

Index (23) can be interpreted as the geometric mean of Laspeyres-like and Paasche-like indices, and it thus has the Fisher form so to speak. Similarly, we can identify the contribution of changes in the real exchange rate as:

$$Z_{E,t,t-1} \equiv \sqrt{\frac{z(h_{t-1}, e_t, v_{K,t-1}, v_{L,t-1}, t-1)}{z(h_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \times \frac{z(h_t, e_t, v_{K,t}, v_{L,t}, t)}{z(h_t, e_{t-1}, v_{K,t}, v_{L,t}, t)}} \,. \tag{24}$$

The contribution of changes in domestic factor endowments is obtained as:

$$Z_{K,t,t-1} \equiv \sqrt{\frac{z(h_{t-1}, e_{t-1}, v_{K,t}, v_{L,t-1}, t-1)}{z(h_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \times \frac{z(h_t, e_t, v_{K,t}, v_{L,t}, t)}{z(h_t, e_t, v_{K,t-1}, v_{L,t}, t)}} \tag{25}$$

$$Z_{L,t,t-1} \equiv \sqrt{\frac{z(h_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t}, t-1)}{z(h_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \times \frac{z(h_t, e_t, v_{K,t}, v_{L,t}, t)}{z(h_t, e_t, v_{K,t}, v_{L,t-1}, t)}} \,, \tag{26}$$

and, finally, the contribution of technological progress:

$$Z_{T,t,t-1} \equiv \sqrt{\frac{z(h_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t)}{z(h_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \times \frac{z(h_t, e_t, v_{K,t}, v_{L,t}, t)}{z(h_t, e_t, v_{K,t}, v_{L,t}, t-1)}} \,. \tag{27}$$

6. Measurement

If the functional form of real GDI function (17) is known, it can be directly substituted into (23)-(27), and the various indices can thus easily be calculated provided that the necessary data are available. One form well suited for our purposes is the Translog function.[15] It provides a second-order approximation in logarithms of an arbitrary real GDI function, and it is as follows:

$$
\begin{aligned}
\ln z_t = {} & \alpha_0 + \alpha_H \ln h_t + \alpha_E \ln e_t + \beta_K \ln v_{K,t} + (1 - \beta_K) \ln v_{L,t} \\
& + \frac{1}{2} \gamma_{HH} (\ln h_t)^2 + \gamma_{HE} \ln h_t \ln e_t + \frac{1}{2} \gamma_{EE} (\ln e_t)^2 \\
& + \frac{1}{2} \phi_{KK} (\ln v_{K,t} - \ln v_{L,t})^2 + (\delta_{HK} \ln h_t + \delta_{EK} \ln e_t)(\ln v_{K,t} - \ln v_{L,t}) \\
& + (\delta_{HT} \ln h_t + \delta_{ET} \ln e_t) t + \phi_{KT} (\ln v_{K,t} - \ln v_{L,t}) t + \beta_T t + \frac{1}{2} \phi_{TT} t^2
\end{aligned}
\tag{28}
$$

In the case of (28) the first-order conditions (18)-(22) can be derived in share form through logarithmic differentiation:

$$
\frac{\partial \ln z(\cdot)}{\partial \ln h} = -s_{A,t} = \alpha_H + \gamma_{HH} \ln h_t + \gamma_{HE} \ln e_t + \delta_{HK} \ln \frac{v_{K,t}}{v_{L,t}} + \delta_{HT} t
\tag{29}
$$

$$
\frac{\partial \ln z(\cdot)}{\partial \ln e} = s_{B,t} = \alpha_E + \gamma_{HE} \ln h_t + \gamma_{EE} \ln e_t + \delta_{EK} \ln \frac{v_{K,t}}{v_{L,t}} + \delta_{ET} t
\tag{30}
$$

$$
\frac{\partial \ln z(\cdot)}{\partial \ln v_K} = s_{K,t} = \beta_K + \delta_{HK} \ln h_t + \delta_{EK} \ln e_t + \phi_{KK} \ln \frac{v_{K,t}}{v_{L,t}} + \phi_{KT} t
\tag{31}
$$

$$
\frac{\partial \ln z(\cdot)}{\partial \ln v_L} = s_{L,t} = 1 - \beta_K - \delta_{HK} \ln h_t - \delta_{EK} \ln e_t - \phi_{KK} \ln \frac{v_{K,t}}{v_{L,t}} - \phi_{KT} t
\tag{32}
$$

$$
\frac{\partial \ln z(\cdot)}{\partial t} = \mu_t = \beta_T + \delta_{HT} \ln h_t + \delta_{ET} \ln e_t + \phi_{KT} \ln \frac{v_{K,t}}{v_{L,t}} + \phi_{TT} t \,,
\tag{33}
$$

where s_A is the average share of foreign trade in GDP ($s_A \equiv \frac{1}{2} s_X + \frac{1}{2} s_M$), s_B is the trade balance relative to GDP ($s_B \equiv s_X - s_M$), and s_K and s_L are the GDP share of capital and labour, respectively.

If the true real GDI function is Translog, it turns out that indices (23)-(27) can be calculated on the basis of the data alone, that is without knowledge of the parameters of (28). Indeed, introducing (28) into (23)-(27) and making use of (29)-(33), one finds:

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[15] The GDP/GNP function has first been applied to the case of an open economy by Kohli (1978). See Woodland (1982) and Kohli (1991) for additional details. See Christensen, Jorgenson, and Lau (1973) and Diewert (1974) for the Translog functional form.

$$Z_{H,t,t-1} = \exp\left[-\frac{1}{2}(s_{A,t} + s_{A,t-1}) \ln \frac{h_t}{h_{t-1}}\right] \tag{34}$$

$$Z_{E,t,t-1} = \exp\left[\frac{1}{2}(s_{B,t} + s_{B,t-1}) \ln \frac{e_t}{e_{t-1}}\right] \tag{35}$$

$$Z_{K,t,t-1} \equiv \exp\left[\frac{1}{2}(s_{K,t} + s_{K,t-1}) \ln \frac{v_{K,t}}{v_{K,t-1}}\right] \tag{36}$$

$$Z_{L,t,t-1} \equiv \exp\left[\frac{1}{2}(s_{L,t} + s_{L,t-1}) \ln \frac{v_{L,t}}{v_{L,t-1}}\right] \tag{37}$$

$$Z_{T,t,t-1} \equiv \frac{Z_{t,t-1}}{V_{t,t-1}}. \tag{38}$$

In this last expression $V_{t,t-1}$ is a Tornqvist index of domestic input quantities:

$$V_{t,t-1} = \exp\left[\frac{s_{K,t} + s_{K,t-1}}{2} \ln \frac{v_{K,t}}{v_{K,t-1}} + \frac{s_{L,t} + s_{L,t-1}}{2} \ln \frac{v_{L,t}}{v_{L,t-1}}\right]. \tag{39}$$

It can now easily be seen that – still assuming that the true real GDI function is indeed given by (28) – indices (34)-(38) together give a *complete* decomposition of the growth in real GDI:[16]

$$Z_{t,t-1} = Z_{H,t,t-1} \times Z_{E,t,t-1} \times Z_{K,t,t-1} \times Z_{L,t,t-1} \times Z_{T,t,t-1}. \tag{40}$$

That is, the actual change in real GDI can be decomposed into five real factors: the terms-of-trade effect, the real exchange-rate effect, the capital endowment effect, the labour endowment effects, and the technological change (also known as total factor productivity) effect.

From (7), (14), (34) and (35), and recalling that $s_N + s_X - s_M = 1$, one finds:

$$P_{t,t-1} = P_{N,t,t-1} \times Z_{H,t,t-1} \times Z_{E,t,t-1} \tag{41}$$

In other words, the Törnqvist GDP price deflator incorporates the terms-of-trade and the real-exchange-rate effects, two effects that should be viewed as real – as opposed to price – effects. Conversely, in view of (9) and (13), (41) implies:

$$Z_{t,t-1} = Y_{t,t-1} \times Z_{H,t,t-1} \times Z_{E,t,t-1} \tag{42}$$

That is, real GDI ($Z_{t,t-1}$) is equal to real GDP ($Y_{t,t-1}$) augmented by the terms-of-trade and real-exchange-rate effects. Put differently, as indicated by (40), real GDP, by excluding $Z_{H,t,t-1}$ and $Z_{E,t,t-1}$, takes account of only three out of five real forces.

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[16] See Kohli (1990, 2004c).

7. Estimates for Hong Kong

We now illustrate our results with data from Hong Kong. The data are annual for the period 1961-2003. They consist of current price and constant price data for the major components of GDP. The domestic price index ($P_{N,t,t-1}$) was obtained as a Törnqvist index of consumption, investment and government purchases. We show in Table 1 estimates of $Q_{t,t-1}, G_{t,t-1}, Y_{t,t-1}, Z_{t,t-1}, Z_{H,t,t-1}$ and $Z_{E,t,t-1}$, based on (5), (6), (9), (13), (34) and (35), respectively.[17] Geometric means are reported at the bottom of the table. The path of real GDI (the cumulated value of $Z_{t,t-1}$) can also be seen in Figure 3. The yearly growth factors of real GDP ($Q_{t,t-1}$) and real GDI ($Z_{t,t-1}$) are shown in Figure 6.

From 1962 to 2003, the official measure of real GDP suggests that real growth has reached 6.8% on average, a truly remarkable performance. The chained Laspeyres quantity index indicates a somewhat lower value, namely 6.3%, whereas the implicit Törnqvist measure yields a value of 6.4%. The fact that the implicit Törnqvist shows a higher value than the chained Laspeyres quantity index is consistent with the fact that in the supply context the latter is biased downwards.

The third column of Table 1 shows estimates of real GDI. It averaged 7.2% over the sample period, substantially more than either measure of real GDP. This difference can be attributed to the gains from trade, i.e. the combined effect of the terms-of-trade and the real-exchange-rate effects. These two effects are shown in the last two columns of the table. The terms-of-trade effect is found to be particularly large: it has added about 0.82% of real growth per year. The real-exchange-rate effect, on the other hand, was very small. Nonetheless, it cannot be ignored if one wants the decomposition (38) to hold exactly. While the real-exchange-rate effect is small on average, it can be quite large for particular observations. Thus, it reduced real value added by as much as 0.5% in 1986, 1988, and 1989, and it added about 0.3% in 1973 and 1974. The terms-of-trade effect too was far from steady. Thus, there are six years where it added more than 3% in growth (1962, 1963, 1971, 1975, 1976, 1980) and three years (1974, 1986, 1995) when it reduced growth by more than 3%. The time profile of $Z_{H,t,t-1}$ and $Z_{E,t,t-1}$ can also be depicted graphically. This is done in Figure 7. The dominating role of the terms-of-trade effect is clearly apparent.

8. Conclusion

To sum up, it seems that Hong Kong's growth performance over the past four decades has been even more stellar than the data suggest. Real growth may have been underestimated by real GDP by about 0.43% per year. Much of the discrepancy originates in the sixties, seventies, and eighties. Moreover, it appears that this average figure masks large positive or negative deviations in particular years. The discrepancy in the growth rates of real GDP and real domestic income consists of four elements: over the sample period, the direct Laspeyres index of real GDP overestimated growth by about 0.40% compared to a Laspeyres chained index, and by an additional 0.04% by ignoring the real-exchange-rate effect. On the other hand, it underestimated growth by about 0.06% by providing a linear, rather

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[17]   If data on the prices and quantities of labour and capital services, estimates of (36)-(38) could readily be computed.

than a superlative measure, and by an additional 0.82% by neglecting the terms-of-trade effect. By compounding these deviations, one arrives at the figure of 0.43% mentioned above. A discrepancy of 0.43% might not seem like much, but compounded over a period of 42 years, it adds up to nearly 20% of GDP.

One could object that even if it is true that real value added and real income increase as a result of an improvement in the terms of trade, this is of limited interest since it does not create a single job. The reason why many economists are interested in GDP figures is because an increase in real GDP is usually associated with an increase in employment. Even if it is true that an improvement in the terms of trade does not necessarily create any additional jobs, this criticism is beside the point for several reasons. For a start, as we showed above, an improvement in the terms of trade leads to a *reduction* in real GDP, a reduction that is totally meaningless. Second, a technological progress, which is integrated in the calculation of real GDP, brings about an increase in real GDP without necessarily raising the demand for labour either. Both phenomena are perfectly similar, and there is therefore no reason to treat them differently. In fact, if one were really interested in the demand for labour, it would be much more sensible to derive it from a GDI function such as (17), instead of relying on a crude and biased indicator such as real GDP. Finally, one ought to remember that it is real income – and ultimately consumption – that generate utility, rather than work effort, which is usually considered to have a negative marginal utility. Work is a means to get an income, it is not a goal in itself.

One could also object that real GDP attempts to measure a country's production effort – production requires hard work – and that there is little merit in experiencing an effortless improvement in the terms of trade. We would strongly disagree with such a narrow view of production and transformation activities. Production implies the transformation of inputs into outputs. In a modern economy, specialisation and the gains from trade play a key role. International trade offers additional opportunities to transform some intermediate goods into others, and it thus makes it possible to expand the production possibilities. While an improvement in the terms of trade may indeed be a purely exogenous event, foreign trade is an activity that requires much effort. Importers and exporters must constantly scout world markets in search for better opportunities, and domestic producers must position themselves to exploit existing comparative advantages, and always be on the lookout for new ones. Similar considerations apply to technological change that may require many efforts, but which too may be the outcome of chance or which might even be imported from abroad. There is therefore no reason to treat these two types of efforts differently, all the more so that it may often be impossible to distinguish between a terms-of-trade improvement and a technological change that would have occurred in the transportation industry, for instance.

While we are not advocating that real GDP be abandoned as a statistical concept, we certainly would argue that real GDI is more relevant from a welfare perspective. Feenstra et al. (2004) argue that countries whose GDP^o is larger than their GDP^e as a result of relatively unfavourable terms of trade are really more productive than their real income suggests. A cynical view would be that these countries are mostly good at producing things people do not want.

The measure of real value added that we propose here is somewhat similar to the Command-Basis GNP indicator that the U.S. Bureau of Economic Analysis has been publishing for the past twenty years.[18] This indicator attempts to take terms-of-trade changes into account in a rather crude way, by deflating nominal exports by the price of imports – rather than by the price of exports – in expression (3) above. Even if this procedure goes in the right direction, it is rather *ad hoc* and it rests on no economic analysis. As a measure of income it also suffers from the drawback that it depends on the position of the trade balance, that is, ultimately, on a savings/consumption decision. Our analysis on the contrary suggests that a preferable approach is to deflate nominal GDP by a domestic price index. While we have a preference for a Törnqvist index, a Fisher index could also be envisaged; the results it would produce would be numerically very close.

Finally, we would like to emphasise that our discussion should in no way be interpreted as a criticism of the statistical unit that computes Hong Kong national accounts data, or of any other agency for that matter. Computing real GDP as a Laspeyres quantity index is standard practice almost everywhere in the world. It just so happens that given the huge terms-of-trade improvements that Hong Kong has enjoyed over the years and in view of its exceptional openness, Hong Kong makes for an interesting case study.

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[18] See Denison (1981).

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Table 1. Real GDP, real GDI, terms-of-trade, and real exchange-rate effects

	$Q_{t,t-1}$	$G_{t,t-1}$	$Y_{t,t-1}$	$Z_{t,t-1}$	$Z_{H,t,t-1}$	$Z_{E,t,t-1}$
1962	1.1415	1.0920	1.0937	1.1479	1.0476	1.0018
1963	1.1570	1.1394	1.1392	1.1768	1.0341	0.9989
1964	1.0857	1.0844	1.0847	1.0915	1.0061	1.0002
1965	1.1448	1.1498	1.1491	1.1674	1.0163	0.9996
1966	1.0172	1.0124	1.0157	1.0373	1.0220	0.9993
1967	1.0169	1.0303	1.0326	1.0563	1.0231	1.0000
1968	1.0333	1.0316	1.0331	1.0658	1.0293	1.0023
1969	1.1129	1.1123	1.1125	1.1188	1.0041	1.0016
1970	1.0918	1.0962	1.0968	1.0947	1.0007	0.9974
1971	1.0708	1.0521	1.0532	1.1027	1.0468	1.0001
1972	1.1033	1.1111	1.1103	1.1202	1.0111	0.9979
1973	1.1236	1.1150	1.1148	1.1124	0.9951	1.0028
1974	1.0233	1.0255	1.0286	0.9782	0.9480	1.0032
1975	1.0033	0.9992	1.0000	1.0304	1.0325	0.9980
1976	1.1623	1.1655	1.1663	1.2111	1.0371	1.0012
1977	1.1173	1.1199	1.1200	1.1138	0.9942	1.0002
1978	1.0850	1.0893	1.0892	1.0913	1.0021	0.9999
1979	1.1152	1.0847	1.0856	1.1030	1.0161	0.9999
1980	1.1021	1.0752	1.0773	1.1106	1.0308	1.0000
1981	1.0924	1.0937	1.0935	1.0765	0.9842	1.0003
1982	1.0279	1.0211	1.0213	1.0339	1.0121	1.0003
1983	1.0580	1.0452	1.0463	1.0450	0.9985	1.0002
1984	1.0999	1.0959	1.0977	1.1292	1.0267	1.0019
1985	1.0051	1.0185	1.0195	1.0432	1.0251	0.9981
1986	1.1078	1.1053	1.1039	1.0589	0.9637	0.9953
1987	1.1302	1.1160	1.1178	1.1317	1.0155	0.9970
1988	1.0803	1.0660	1.0667	1.0662	1.0047	0.9949
1989	1.0265	1.0190	1.0195	1.0415	1.0263	0.9953
1990	1.0370	1.0334	1.0341	1.0368	1.0062	0.9964
1991	1.0564	1.0491	1.0509	1.0641	1.0162	0.9965
1992	1.0659	1.0718	1.0735	1.0859	1.0146	0.9969
1993	1.0635	1.0556	1.0549	1.0626	1.0114	0.9959
1994	1.0548	1.0667	1.0667	1.0526	0.9882	0.9985
1995	1.0389	1.0437	1.0412	1.0099	0.9697	1.0003
1996	1.0431	1.0431	1.0427	1.0547	1.0093	1.0021
1997	1.0507	1.0504	1.0514	1.0565	1.0027	1.0022
1998	0.9503	0.9438	0.9441	0.9542	1.0100	1.0008
1999	1.0342	1.0270	1.0268	1.0124	0.9857	1.0002
2000	1.1016	1.0942	1.0938	1.0816	0.9869	1.0019
2001	1.0046	1.0046	1.0049	1.0112	1.0065	0.9998
2002	1.0189	1.0193	1.0201	1.0353	1.0139	1.0011
2003	1.0322	1.0342	1.0337	1.0148	0.9798	1.0020
1962-2003	**1.0677**	**1.0634**	**1.0640**	**1.0723**	**1.0082**	**0.9996**

Figure 1. Terms of trade and real exchange rate
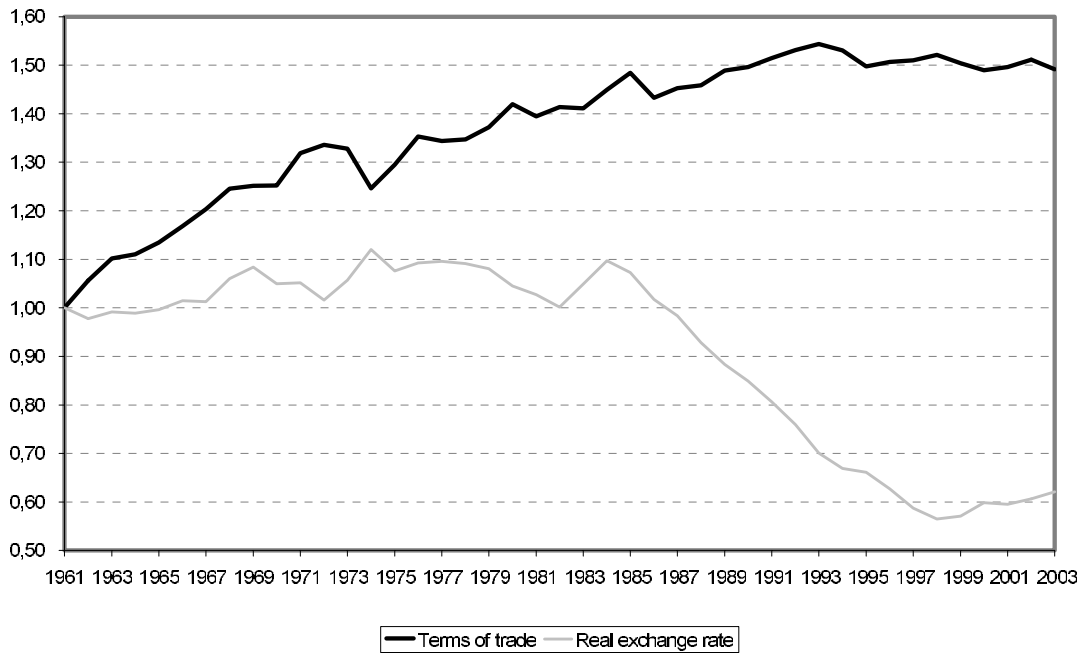
Note: The real exchange rate is defined as the price of tradables relative to the price of nontradables; see expression (12). A fall in e indicates a real appreciation of the HK dollar.

Figure 2. GDP shares of exports and imports
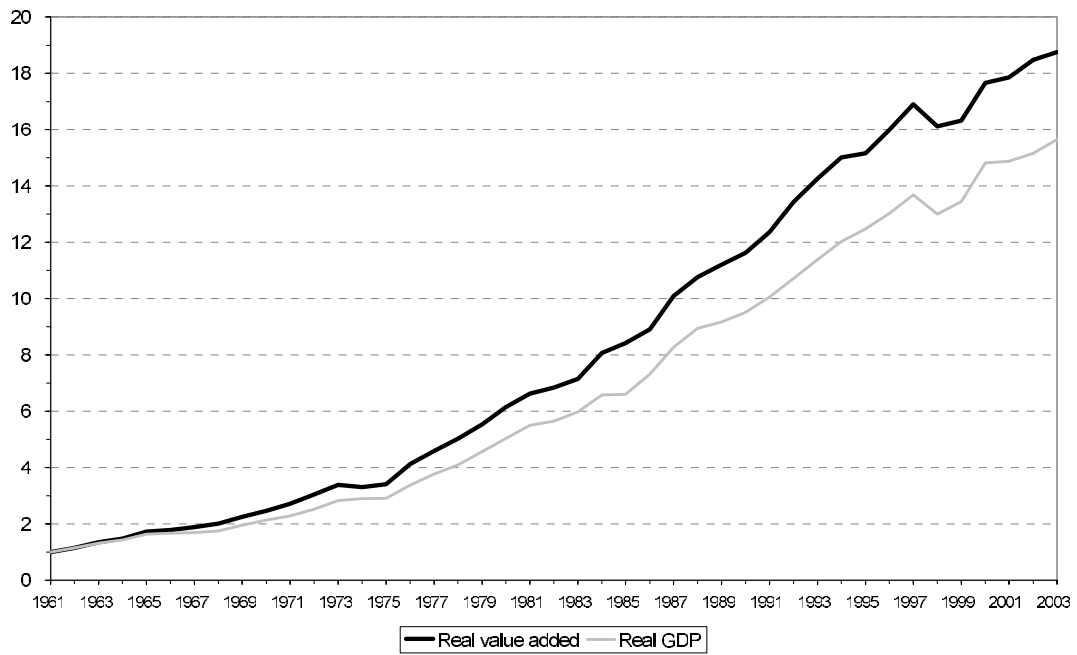
Figure 3. Real value added and real GDP

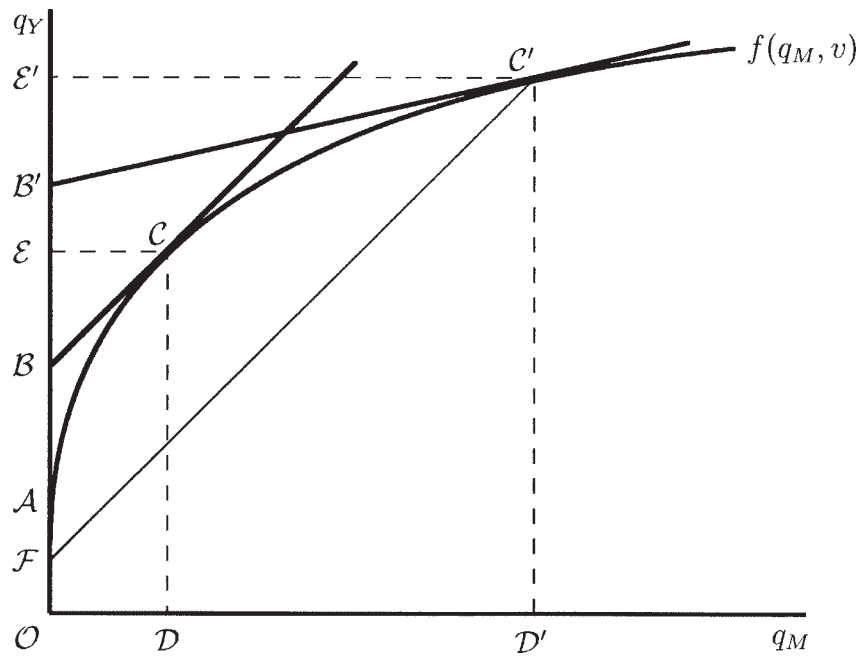

Figure 4. Imports as an input to the technology
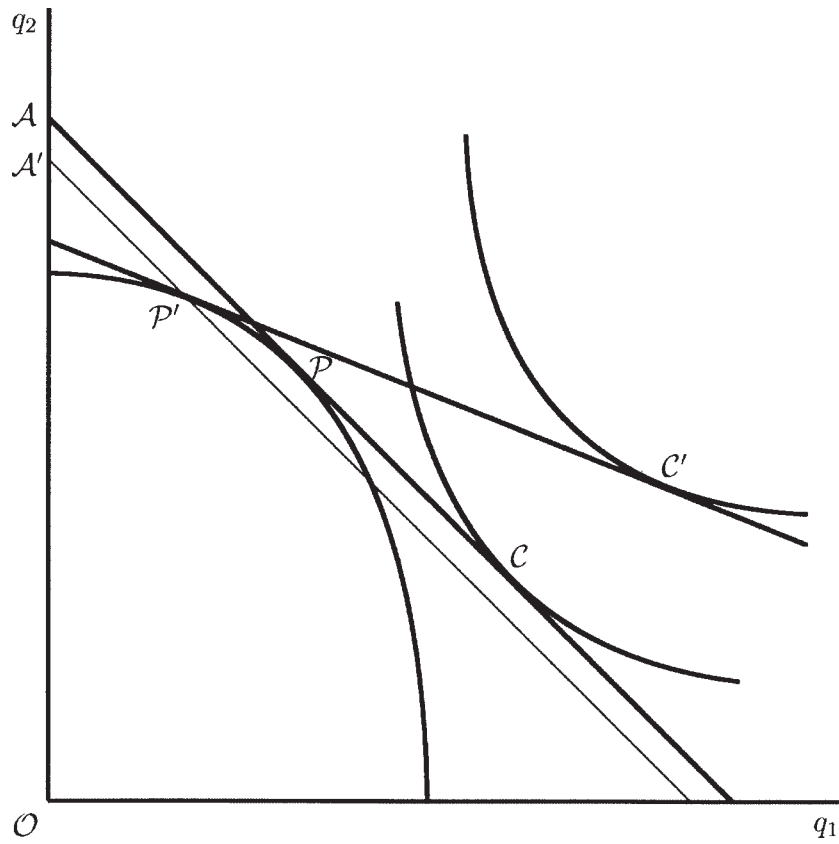
Figure 5. Trade in final goods

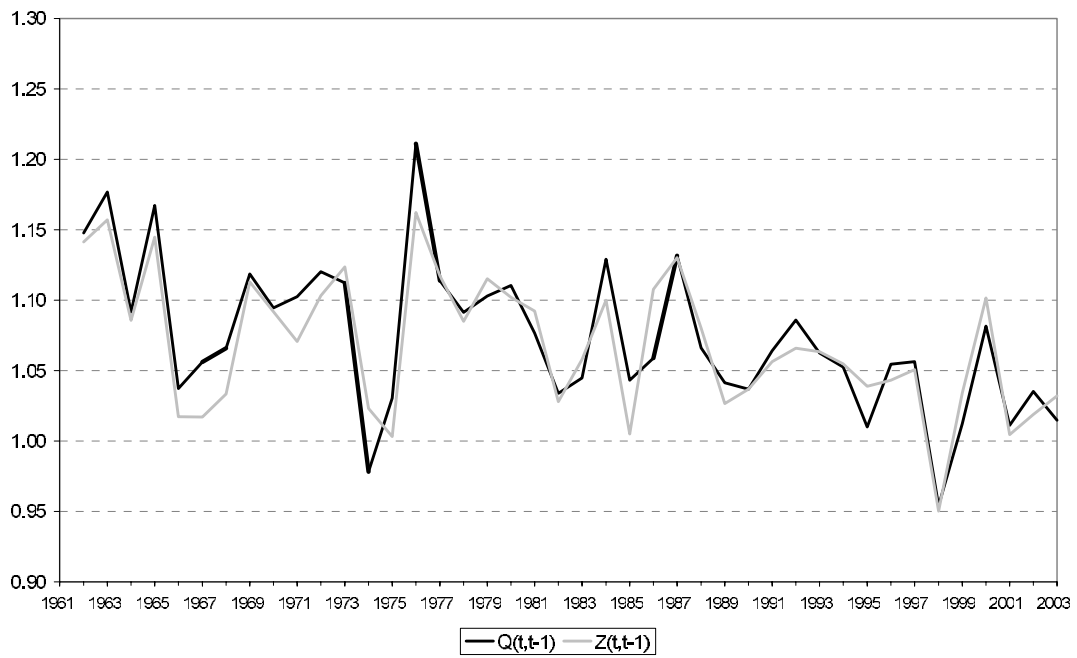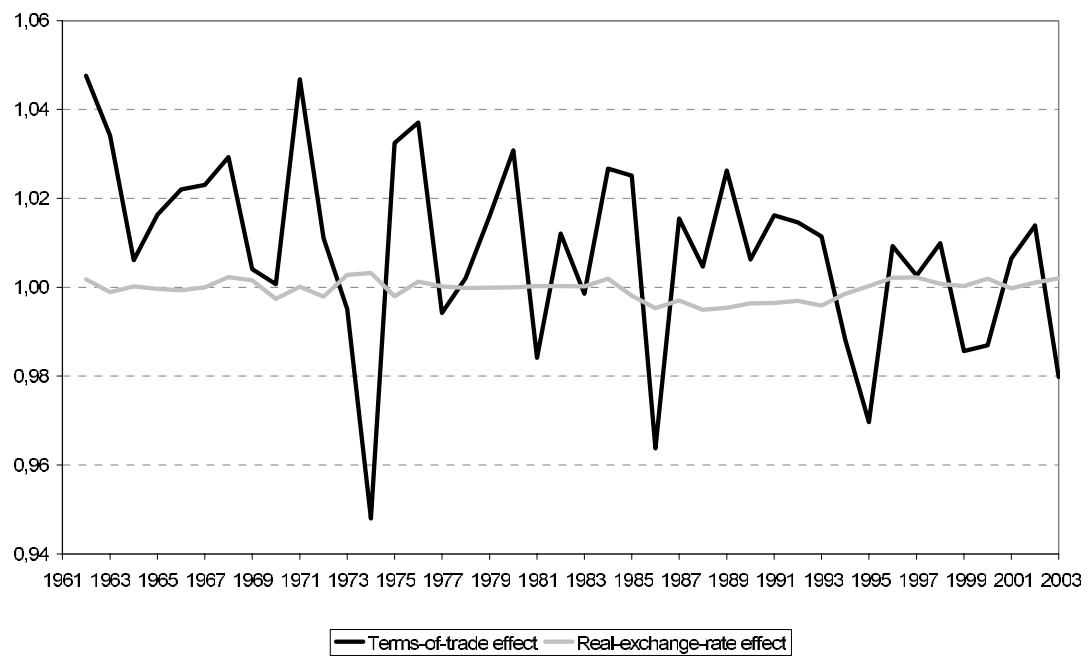**Figure 6. Real GDP and real GDI (growth factors)**

Figure 7. Terms-of-trade and real-exchange-rate effects

Appendix

The purpose of this appendix is to derive expressions (18) – (22) of the main text. Let $\pi(p_M, p_X, p_N, v_K, v_L, t)$ be the country's nominal GDP function. It is defined as follows (see Kohli, 1978, 1991; the time subscripts are omitted for simplicity):

$$\pi(p_M, p_X, p_N, v_K, v_L, t) \equiv \max_{q_N, q_X, q_M} \left\{ \begin{array}{c} p_N q_N + p_X q_X - p_M q_M : \\ (q_N, q_X, q_M, v_K, v_L) \in T \end{array} \right\}. \tag{A1}$$

From (17) it can easily be seen that:

$$\pi(p_M, p_X, p_N, v_K, v_L, t) = p_N z(h, e, v_K, v_L, t). \tag{A2}$$

The slope properties of $\pi(p_M, p_X, p_N, v_K, v_L, t)$ are well known (Kohli, 1978, 1991):

$$\frac{\partial \pi(\cdot)}{\partial p_M} = -q_M \tag{A3}$$

$$\frac{\partial \pi(\cdot)}{\partial p_X} = q_X \tag{A4}$$

$$\frac{\partial \pi(\cdot)}{\partial p_N} = q_N \tag{A5}$$

$$\frac{\partial \pi(\cdot)}{\partial v_K} = w_K \tag{A6}$$

$$\frac{\partial \pi(\cdot)}{\partial v_L} = w_L \tag{A7}$$

$$\frac{\partial \pi(\cdot)}{\partial t} = \mu \pi. \tag{A8}$$

Making use of (A2) these conditions can be rewritten as:

$$\frac{\partial \pi(\cdot)}{\partial p_M} = p_N \frac{dz(\cdot)}{dp_M} = p_N \left(z_h \frac{1}{p_X} + \frac{1}{2} z_e \frac{p_M^{-1/2} p_X^{1/2}}{p_N} \right) = \frac{p_N}{p_M} \left(hz_h + \frac{1}{2} ez_e \right) = -q_M \tag{A9}$$

$$\frac{\partial \pi(\cdot)}{\partial p_X} = p_N \frac{dz(\cdot)}{dp_X} = p_N \left(-z_h \frac{p_M}{p_X^2} + \frac{1}{2} z_e \frac{p_M^{1/2} p_X^{-1/2}}{p_N} \right) = \frac{p_N}{p_X} \left(-hz_h + \frac{1}{2} ez_e \right) = q_X \tag{A10}$$

$$\frac{\partial \pi(\cdot)}{\partial p_N} = z + p_N \frac{dz(\cdot)}{dp_N} = z - z_e \frac{p_M^{1/2} p_X^{1/2}}{p_N} = z - ez_e = q_N \tag{A11}$$

$$\frac{\partial \pi(\cdot)}{\partial v_K} = p_N \frac{dz(\cdot)}{dv_K} = p_N z_K = w_K \qquad\qquad (A12)$$

$$\frac{\partial \pi(\cdot)}{\partial v_L} = p_N \frac{dz(\cdot)}{dv_L} = p_N z_L = w_L \qquad\qquad (A13)$$

$$\frac{\partial \pi(\cdot)}{\partial t} = p_N \frac{dz(\cdot)}{dt} = p_N z_t = \mu\, p_N z, \qquad\qquad (A14)$$

where $z_h \equiv \partial z(\cdot)/\partial h$, $z_e \equiv \partial z(\cdot)/\partial e$, and so on. Jointly solving (A9) and (A10) for z_h and z_e we find:

$$z_h = \frac{-p_X q_X - p_M q_M}{2 h p_N} = -\frac{1}{2}\frac{e}{h}\left(h^{-\frac{1}{2}} q_X + h^{\frac{1}{2}} q_M\right) \qquad\qquad (A15)$$

$$z_e = \frac{p_X q_X - p_M q_M}{e p_N} = h^{-\frac{1}{2}} q_X - h^{\frac{1}{2}} q_M \qquad\qquad (A16)$$

i.e. expressions (18) and (19). Expressions (20) – (22) follow immediately from (A12) – (A14).